SOLUTION As noted above, each spring stretches by an amount given by $x = F/k$ (Equation 10.1). Therefore, the difference d in the heights between the high and low ends of the rod is

$$d = x_1 - x_2 = \frac{F_1}{k_1} - \frac{F_2}{k_2}$$

Now, using the fact that both force magnitudes are equal to half the weight of the rod $\left(F_1 = F_2 = \frac{1}{2}mg\right)$, we obtain

$$d = \frac{\frac{1}{2}mg}{k_1} - \frac{\frac{1}{2}mg}{k_2} = \frac{mg}{2}\left(\frac{1}{k_1} - \frac{1}{k_2}\right)$$

Thus, using Equation 1.4 for the angle that the rod makes with the horizontal, we find

$$\theta = \sin^{-1}\left(\frac{d}{L}\right) = \sin^{-1}\left[\frac{mg}{2L}\left(\frac{1}{k_1} - \frac{1}{k_2}\right)\right]$$

$$= \sin^{-1}\left[\frac{(1.4\text{ kg})(9.80\text{ m/s}^2)}{2(0.75\text{ m})}\left(\frac{1}{33\text{ N/m}} - \frac{1}{59\text{ N/m}}\right)\right] = \boxed{7.0°}$$

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9. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The force that acts on the block is given by Newton's Second law, $F_x = ma_x$ (Equation 4.2a). Since the block has a constant acceleration, the acceleration is given by Equation 2.8 with $v_0 = 0$ m/s; that is, $a_x = 2d/t^2$, where d is the distance through which the block is pulled. Therefore, the force that acts on the block is given by

$$F_x = ma_x = \frac{2md}{t^2}$$

The force acting on the block is the restoring force of the spring. Thus, according to Equation 10.2, $F_x = -kx$, where k is the spring constant and x is the displacement. Solving Equation 10.2 for x and using the expression above for F_x, we obtain

$$x = -\frac{F_x}{k} = -\frac{2md}{kt^2} = -\frac{2(7.00\text{ kg})(4.00\text{ m})}{(415\text{ N/m})(0.750\text{ s})^2} = -0.240\text{ m}$$

The amount that the spring stretches is $\boxed{0.240\text{ m}}$.

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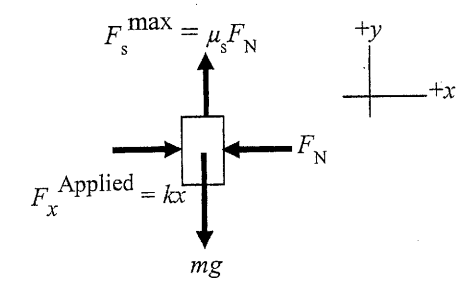
10. ***REASONING*** The free-body diagram shows the magnitudes and directions of the forces acting on the block. The weight mg acts downward. The maximum force of static friction f_s^{max} acts upward just before the block begins to slip. The force from the spring $F_x^{\text{Applied}} = kx$ (Equation 10.1) is directed to the right. The normal force F_N from the wall points to the left. The magnitude of the maximum force of static friction is related to the magnitude of the normal force according to Equation 4.7 $\left(f_s^{\text{max}} = \mu_s F_N\right)$, where μ_s is the coefficient of static friction. Since the block is at equilibrium just before it begins to slip, the forces in the x direction must balance and the forces in the y direction must balance. The balance of forces in the two directions will provide two equations, from which we will determine the coefficient of static friction.



SOLUTION Since the forces in the x direction and in the y direction must balance, we have

$$F_N = kx \quad \text{and} \quad mg = \mu_s F_N$$

Substituting the first equation into the second equation gives

$$mg = \mu_s F_N = \mu_s(kx) \quad \text{or} \quad \mu_s = \frac{mg}{kx} = \frac{(1.6\text{ kg})(9.80\text{ m/s}^2)}{(510\text{ N/m})(0.039\text{ m})} = \boxed{0.79}$$

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11. $\boxed{\text{SSM}}$ ***REASONING*** When the ball is whirled in a horizontal circle of radius r at speed v, the centripetal force is provided by the restoring force of the spring. From Equation 5.3, the magnitude of the centripetal force is mv^2/r, while the magnitude of the restoring force is kx (see Equation 10.2). Thus,

$$\frac{mv^2}{r} = kx \tag{1}$$

The radius of the circle is equal to $(L_0 + \Delta L)$, where L_0 is the unstretched length of the spring and ΔL is the amount that the spring stretches. Equation (1) becomes

$$\frac{mv^2}{L_0 + \Delta L} = k\Delta L \tag{1'}$$

c. The maximum speed v_{max} is the product of the amplitude and the angular frequency:

$$v_{max} = A\omega = (0.120 \text{ m})(10.5 \text{ rad/s}) = \boxed{1.26 \text{ m/s}} \quad (10.8)$$

d. The magnitude a_{max} of the maximum acceleration is

$$a_{max} = A\omega^2 = (0.120 \text{ m})(10.5 \text{ rad/s})^2 = \boxed{13.2 \text{ m/s}^2} \quad (10.10)$$

18. ***REASONING AND SOLUTION***
a. Since the object oscillates between ± 0.080 m, the motion's amplitude of the motion is $\boxed{0.080 \text{ m}}$.

b. From the graph, the period is $T = 4.0$ s. Therefore, according to Equation 10.4,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4.0 \text{ s}} = \boxed{1.6 \text{ rad/s}}$$

c. Equation 10.11 relates the angular frequency to the spring constant: $\omega = \sqrt{k/m}$. Solving for k we find

$$k = \omega^2 m = (1.6 \text{ rad/s})^2 (0.80 \text{ kg}) = \boxed{2.0 \text{ N/m}}$$

d. At $t = 1.0$ s, the graph shows that the spring has its maximum displacement. At this location, the object is momentarily at rest, so that its speed is $\boxed{v = 0 \text{ m/s}}$.

e. The acceleration of the object at $t = 1.0$ s is a maximum, and its magnitude is

$$a_{max} = A\omega^2 = (0.080 \text{ m})(1.6 \text{ rad/s})^2 = \boxed{0.20 \text{ m/s}^2}$$

19. ***REASONING*** The amplitude of simple harmonic motion is the distance from the equilibrium position to the point of maximum height. The angular frequency ω is related to the period T of the motion by Equation 10.6. The maximum speed attained by the person is the product of the amplitude and the angular speed (Equation 10.8).

SOLUTION
a. Since the distance from the equilibrium position to the point of maximum height is the amplitude A of the motion, we have that $A = 45.0 \text{ cm} = \boxed{0.450 \text{ m}}$.

b. The angular frequency is inversely proportional to the period of the motion:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.90 \text{ s}} = \boxed{3.31 \text{ rad/s}} \quad (10.6)$$

c. The maximum speed v_{max} attained by the person on the trampoline depends on the amplitude A and the angular frequency ω of the motion:

$$v_{max} = A\omega = (0.450 \text{ m})(3.31 \text{ rad/s}) = \boxed{1.49 \text{ m/s}} \quad (10.8)$$

20. ***REASONING*** The spring constant k of either spring is related to the mass m of the object attached to it and the angular frequency ω of its oscillation by $\omega = \sqrt{k/m}$ (Equation 10.11). Squaring both sides of Equation 10.11 and solving for the mass m, we find that $m = k/\omega^2$. The masses m of the two objects are unknown but identical, so we eliminate m and obtain

$$m = \frac{k_1}{\omega_1^2} = \frac{k_2}{\omega_2^2} \quad \text{or} \quad k_2 = \frac{k_1\omega_2^2}{\omega_1^2} \quad (1)$$

To deal with the angular frequencies, we turn to the maximum velocities. The magnitude v_{max} of the maximum velocity of either object is given by $v_{max} = A\omega$ (Equation 10.8), where A is the amplitude of the object's motion. Both objects have a maximum velocity of the same magnitude, so we see that

$$v_{max} = A_1\omega_1 = A_2\omega_2 \quad \text{or} \quad \omega_2 = \frac{A_1\omega_1}{A_2} \quad (2)$$

SOLUTION The amplitude of the motion of the mass attached to spring 1 is twice that of the motion of the mass attached to spring 2: $A_1 = 2A_2$. With this substitution, Equation (2) becomes

$$\omega_2 = \frac{2\cancel{A_2}\omega_1}{\cancel{A_2}} = 2\omega_1 \quad (3)$$

Substituting Equation (3) into Equation (1) yields

$$k_2 = \frac{k_1\omega_2^2}{\omega_1^2} = \frac{k_1(2\cancel{\omega_1})^2}{\cancel{\omega_1^2}} = 4k_1 = 4(174 \text{ N/m}) = \boxed{696 \text{ N/m}}$$

28. ***REASONING*** The elastic potential energy of an ideal spring is given by Equatio[n] $\text{PE}_{\text{elastic}} = \frac{1}{2}ky^2$, where k is the spring constant and y is the amount of str[etch or] compression from the spring's unstrained length. Thus, we need to determine x for an [object] of mass m hanging stationary from the spring. Since the object is stationary, it h[as no] acceleration and is at equilibrium. The upward pull of the spring balances the down[ward-] acting weight of the object. We can use this fact to determine y.

SOLUTION The magnitude of the pull of the spring is given by ky, according to Equ[ation] 10.2. The weight of the object is mg. Since these two forces must balance, we [have] $ky = mg$ or $y = mg/k$. Substituting this result into Equation 10.13 for the elastic poten[tial] energy gives

$$\text{PE}_{\text{elastic}} = \tfrac{1}{2}ky^2 = \tfrac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{m^2g^2}{2k}$$

Applying this expression to the two spring/object systems, we find

$$\text{PE}_1 = \frac{m_1^2g^2}{2k} \quad \text{and} \quad \text{PE}_2 = \frac{m_2^2g^2}{2k}$$

Here, we have omitted the subscript "elastic" for convenience. Dividing the right-hand equation by the left-hand equation gives

$$\frac{\text{PE}_2}{\text{PE}_1} = \frac{\dfrac{m_2^2g^2}{2k}}{\dfrac{m_1^2g^2}{2k}} = \frac{m_2^2}{m_1^2} \quad \text{or} \quad \text{PE}_2 = \text{PE}_1\left(\frac{m_2^2}{m_1^2}\right) = (1.8\text{ J})\frac{(5.0\text{ kg})^2}{(3.2\text{ kg})^2} = \boxed{4.4\text{ J}}$$

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29. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** If we neglect air resistance, only the conservative forces of the spring and gravity act on the object. Therefore, the principle of conservation of mechanical energy applies.

When the 2.00 kg object is hung on the end of the vertical spring, it stretches the spring by an amount y, where

$$y = \frac{F}{k} = \frac{mg}{k} = \frac{(2.00\text{ kg})(9.80\text{ m/s}^2)}{50.0\text{ N/m}} = 0.392\text{ m} \qquad (10.1)$$

This position represents the equilibrium position of the system with the 2.00-kg object suspended from the spring. The object is then pulled down another 0.200 m and released from rest ($v_0 = 0$ m/s). At this point the spring is stretched by an amount of

$0.392\text{ m} + 0.200\text{ m} = 0.592\text{ m}$. This point represents the zero reference level ($h = 0$ m) for the gravitational potential energy.

h = 0 m: The kinetic energy, the gravitational potential energy, and the elastic potential energy at the point of release are:

$$\text{KE} = \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}m(0\text{ m/s})^2 = \boxed{0\text{ J}}$$

$$\text{PE}_{\text{gravity}} = mgh = mg(0\text{ m}) = \boxed{0\text{ J}}$$

$$\text{PE}_{\text{elastic}} = \tfrac{1}{2}ky_0^2 = \tfrac{1}{2}(50.0\text{ N/m})(0.592\text{ m})^2 = \boxed{8.76\text{ J}}$$

The total mechanical energy E_0 at the point of release is the sum of the three energies above: $E_0 = \boxed{8.76\text{ J}}$.

h = 0.200 m: When the object has risen a distance of $h = 0.200$ m above the release point, the spring is stretched by an amount of $0.592\text{ m} - 0.200\text{ m} = 0.392\text{ m}$. Since the total mechanical energy is conserved, its value at this point is still $E = \boxed{8.76\text{ J}}$. The gravitational and elastic potential energies are:

$$\text{PE}_{\text{gravity}} = mgh = (2.00\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) = \boxed{3.92\text{ J}}$$

$$\text{PE}_{\text{elastic}} = \tfrac{1}{2}ky^2 = \tfrac{1}{2}(50.0\text{ N/m})(0.392\text{ m})^2 = \boxed{3.84\text{ J}}$$

Since $\text{KE} + \text{PE}_{\text{gravity}} + \text{PE}_{\text{elastic}} = E$,

$$\text{KE} = E - \text{PE}_{\text{gravity}} - \text{PE}_{\text{elastic}} = 8.76\text{ J} - 3.92\text{ J} - 3.84\text{ J} = \boxed{1.00\text{ J}}$$

h = 0.400 m: When the object has risen a distance of $h = 0.400$ m above the release point, the spring is stretched by an amount of $0.592\text{ m} - 0.400\text{ m} = 0.192\text{ m}$. At this point, the total mechanical energy is still $E = \boxed{8.76\text{ J}}$. The gravitational and elastic potential energies are:

$$\text{PE}_{\text{gravity}} = mgh = (2.00\text{ kg})(9.80\text{ m/s}^2)(0.400\text{ m}) = \boxed{7.84\text{ J}}$$

$$\text{PE}_{\text{elastic}} = \tfrac{1}{2}ky^2 = \tfrac{1}{2}(50.0\text{ N/m})(0.192\text{ m})^2 = \boxed{0.92\text{ J}}$$

The kinetic energy is

$$\mathrm{KE} = E - \mathrm{PE}_{\text{gravity}} - \mathrm{PE}_{\text{elastic}} = 8.76\ \mathrm{J} - 7.84\ \mathrm{J} - 0.92\ \mathrm{J} = \boxed{0\ \mathrm{J}}$$

The results are summarized in the table below:

h	KE	$\mathrm{PE}_{\text{grav}}$	$\mathrm{PE}_{\text{elastic}}$	E
0 m	0 J	0 J	8.76 J	8.76 J
0.200 m	1.00 J	3.92 J	3.84 J	8.76 J
0.400 m	0.00 J	7.84 J	0.92 J	8.76 J

30. ***REASONING*** Since air resistance is negligible, we can apply the principle of conservation of mechanical energy, which indicates that the total mechanical energy of the block and the spring is the same at the instant it comes to a momentary halt on the spring and at the instant the block is dropped. Gravitational potential energy is one part of the total mechanical energy, and Equation 6.5 indicates that it is mgh for a block of mass m located at a height h relative to an arbitrary zero level. This dependence on h will allow us to determine the height at which the block was dropped.

SOLUTION The conservation of mechanical energy states that the final total mechanical energy E_f is equal to the initial total mechanical energy E_0. The expression for the total mechanical energy for an object on a spring is given by Equation 10.14, so that we have

$$\underbrace{\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f + \tfrac{1}{2}ky_f^2}_{E_f} = \underbrace{\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgh_0 + \tfrac{1}{2}ky_0^2}_{E_0}$$

The block does not rotate, so the angular speeds ω_f and ω_0 are zero. Since the block comes to a momentary halt on the spring and is dropped from rest, the translational speeds v_f and v_0 are also zero. Because the spring is initially unstrained, the initial displacement y_0 of the spring is likewise zero. Thus, the above expression can be simplified as follows:

$$mgh_f + \tfrac{1}{2}ky_f^2 = mgh_0$$

The block was dropped at a height of $h_0 - h_f$ above the compressed spring. Solving the simplified energy-conservation expression for this quantity gives

$$h_0 - h_f = \frac{ky_f^2}{2mg} = \frac{(450\ \mathrm{N/m})(0.025\ \mathrm{m})^2}{2(0.30\ \mathrm{kg})(9.80\ \mathrm{m/s^2})} = \boxed{0.048\ \mathrm{m\ or\ 4.8\ cm}}$$

REASONING Assuming that friction and air resistance are negligible, we can apply the principle of conservation of mechanical energy, which indicates that the total mechanical energy of the rod (or ram) and the spring is the same at the instant it contacts the staple and at the instant the spring is released. Kinetic energy $\frac{1}{2}mv^2$ is one part of the total mechanical energy and depends on the mass m and the speed v of the rod. The dependence on the speed will allow us to determine the speed of the ram at the instant of contact with the staple.

SOLUTION The conservation of mechanical energy states that the final total mechanical energy E_f is equal to the initial total mechanical energy E_0. The expression for the total mechanical energy for a spring/mass system is given by Equation 10.14, so that we have

$$\underbrace{\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f + \tfrac{1}{2}ky_f^2}_{E_f} = \underbrace{\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgh_0 + \tfrac{1}{2}ky_0^2}_{E_0}$$

Since the ram does not rotate, the angular speeds ω_f and ω_0 are zero. Since the ram is initially at rest, the initial translational speed v_0 is also zero. Thus, the above expression can be simplified as follows:

$$\tfrac{1}{2}mv_f^2 + mgh_f + \tfrac{1}{2}ky_f^2 = mgh_0 + \tfrac{1}{2}ky_0^2$$

In falling from its initial height of h_0 to its final height of h_f, the ram falls through a distance of $h_0 - h_f = 0.022$ m, since the spring is compressed 0.030 m from its unstrained length to begin with and is still compressed 0.008 m when the ram makes contact with the staple. Solving the simplified energy-conservation expression for the final speed v_f gives

$$v_f = \sqrt{\frac{k\left(y_0^2 - y_f^2\right)}{m} + 2g\left(h_0 - h_f\right)}$$

$$= \sqrt{\frac{(32\ 000\ \mathrm{N/m})\left[(0.030\ \mathrm{m})^2 - (0.008\ \mathrm{m})^2\right]}{0.140\ \mathrm{kg}} + 2(9.80\ \mathrm{m/s^2})(0.022\ \mathrm{m})} = \boxed{14\ \mathrm{m/s}}$$

32. ***REASONING*** Since air resistance is negligible, we can apply the principle of conservation of mechanical energy, which indicates that the total mechanical energy of the pellet/spring system is the same when the pellet comes to a momentary halt at the top of its trajectory as it is when the pellet is resting on the compressed spring. The fact that the total mechanical energy is conserved will allow us to determine the spring constant.

SOLUTION The conservation of mechanical energy states that the final total mechanical energy E_f is equal to the initial total mechanical energy E_0. The expression for the total mechanical energy for an object on a spring is given by Equation 10.14, so that we have

$$\underbrace{\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f + \tfrac{1}{2}ky_f^2}_{E_f} = \underbrace{\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgh_0 + \tfrac{1}{2}ky_0^2}_{E_0} \quad (1)$$

The pellet does not rotate, so the angular speeds ω_f and ω_0 are zero. Since the pellet is at rest as it sits on the spring and since the pellet comes to a momentary halt at the top of its trajectory, the translational speeds v_0 and v_f are also zero. Because the spring is unstrained when the pellet reaches its maximum height, the final displacement y_f of the spring is likewise zero. Thus, Equation (1) simplifies to :

$$mgh_f = mgh_0 + \tfrac{1}{2}ky_0^2$$

Solving this simplified energy-conservation expression for the spring constant k and noting that the pellet rises to distance of $h_f - h_i = 6.10$ m above its position on the compressed spring, we find that

$$k = \frac{2mg(h_f - h_0)}{y_0^2} = \frac{2(2.10\times10^{-2}\text{ kg})(9.80\text{ m/s}^2)(6.10\text{ m})}{(9.10\times10^{-2}\text{ m})^2} = \boxed{303\text{ N/m}}$$

33. SSM ***REASONING*** The only force that acts on the block along the line of motion is the force due to the spring. Since the force due to the spring is a conservative force, the principle of conservation of mechanical energy applies. Initially, when the spring is unstrained, all of the mechanical energy is kinetic energy, $(1/2)mv_0^2$. When the spring is fully compressed, all of the mechanical energy is in the form of elastic potential energy, $(1/2)kx_{max}^2$, where x_{max}, the maximum compression of the spring, is the amplitude A. Therefore, the statement of energy conservation can be written as

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}kA^2$$

This expression may be solved for the amplitude A.

SOLUTION Solving for the amplitude A, we obtain

$$A = \sqrt{\frac{mv_0^2}{k}} = \sqrt{\frac{(1.00\times10^{-2}\text{ kg})(8.00\text{ m/s})^2}{124\text{ N/m}}} = \boxed{7.18\times10^{-2}\text{ m}}$$

34. ***REASONING*** It is assumed in Example 16 that the only forces acting on the jumper are the gravitational force (his weight) and, for the latter part of his descent, the elastic force of the bungee cord. Therefore, only conservative forces are present, and we may use energy conservation to guide our solution. He possesses gravitational potential energy with respect to the water and elastic potential energy. At the lowest point in his fall, he has no kinetic energy since he comes to a momentary halt and has zero speed.

SOLUTION The conservation of mechanical energy states that the final total mechanical energy E_f is equal to the initial total mechanical energy E_0, or $E_f = E_0$ (Equation 6.9a). The expression for the total mechanical energy of an object is given by Equation 10.14. Thus, the conservation of total mechanical energy can be written as

$$\underbrace{\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f + \tfrac{1}{2}ky_f^2}_{E_f} = \underbrace{\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgh_0 + \tfrac{1}{2}ky_0^2}_{E_0}$$

We can simplify this equation by noting which variables are zero. The jumper starts from rest and momentarily comes to a halt at the bottom of the jump; thus, $v_0 = v_f = 0$ m/s. He does not rotate, so his angular speed is zero; $\omega_f = \omega_0 = 0$ rad/s. Initially, the bungee cord is unstretched, so that $y_0 = 0$ m. Setting these terms to zero in the equation above gives

$$mgh_f + \tfrac{1}{2}ky_f^2 = mgh_0$$

At his lowest point, the bungee cord is stretched in the downward direction, which is taken to be the negative direction. Thus, the stretch y_f is negative, and we note from Figure 10.36 that $y_f = h_f - h_A$, where h_A is 37.0 m. Substituting this expression for y_f into the equation above and rearranging terms, we find that

$$h_f^2 + \underbrace{\left(\frac{2mg}{k} - 2h_A\right)}_{b} h_f + \underbrace{h_A^2 - \frac{2mgh_0}{k}}_{c} = 0$$

$$\text{where } b = \frac{2(68.0\text{ kg})(9.80\text{ m/s}^2)}{66.0\text{ N/m}} - 2(37.0\text{ m}) = -53.8\text{ m}$$

$$\text{and} \quad c = (37.0\text{ m})^2 - \frac{2(68.0\text{ kg})(9.80\text{ m/s}^2)(46.0\text{ m})}{66.0\text{ N/m}} = 440.1\text{ m}^2$$

Thus, we have that

$$h_f^2 - (53.8\text{ m})h_f + (440.1\text{ m}^2) = 0$$

This is a quadratic equation in the variable h_f, and its solution is

$$h_f = \frac{-(-53.8\text{ m}) \pm \sqrt{(-53.8\text{ m})^2 - 4(440.1\text{ m}^2)}}{2} = 45.2\text{ m} \quad \text{or} \quad 10.1\text{ m}$$

The 45.2-m answer is discarded, because it implies that the jumper comes to a halt at a distance of only 46.0 m − 45.2 m = 0.81 m below the platform, which is above the point where the bungee cord is stretched. Thus, when he reaches the lowest point in his fall, his height above the water is $\boxed{10.1\text{ m}}$.

35. ***REASONING*** Since the surface is frictionless, we can apply the principle of conservation of mechanical energy, which indicates that the final total mechanical energy E_f of the object and spring is equal to their initial total mechanical energy E_0; $E_f = E_0$. This conservation equation, and the fact that the angular frequency ω of the oscillation is related to the spring constant k and mass m by $\omega = \sqrt{k/m}$ (Equation 10.11), will permit us to find the speed of the object at the instant when the spring is stretched by 0.048 m.

SOLUTION Equating the total mechanical energy of the system at the instant the spring is stretched by $x_f = +0.048$ m to the total mechanical energy when the spring is compressed by $x_0 = -0.065$ m, we have

$$\underbrace{\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f + \tfrac{1}{2}kx_f^2}_{E_f} = \underbrace{\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgh_0 + \tfrac{1}{2}kx_0^2}_{E_0} \quad (1)$$

The object does not rotate, so the angular speeds ω_f and ω_0 are zero. Since the object is initially at rest, $v_0 = 0$ m/s. Finally, we note that the height of the object does not change during the motion, so $h_f = h_0$. Thus, Equation (1) simplifies to

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}kx_f^2 = \tfrac{1}{2}kx_0^2$$

Solving this expression for the final speed gives

$$v_f = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x_f^2}$$

We now recognize that the term $\sqrt{k/m}$ is the angular frequency ω of the motion (see Equation 10.11). With this substitution, the final speed becomes

$$v_f = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x_f^2} = \omega\sqrt{x_0^2 - x_f^2} = (11.3\text{ rad/s})\sqrt{(-0.065\text{ m})^2 - (0.048\text{ m})^2} = \boxed{0.50\text{ m/s}}$$

36. ***REASONING***

a. As the block rests stationary in its equilibrium position, it has no acceleration. According to Newton's second law, the net force acting on the block is, therefore, zero. This means that the downward-directed weight of the block must be balanced by an upward-directed force. This upward force is the restoring force of the spring and is produced because the spring is compressed. The compression must be enough for the spring to exert on the block a restoring force that has a magnitude equal to the block's weight. This balancing of forces will allow us to determine the magnitude of the spring's compression.

b. As the block falls downward after being released, its speed is changing in the manner characteristic of simple harmonic motion. The block is not in equilibrium, and the forces acting on it do not balance to zero. Instead of thinking about forces, we may think about mechanical energy and its conservation. When the block is released from rest, the energy of the spring/block system is all in the form of gravitational potential energy. Being at rest, the block has no initial kinetic energy. It also has no initial elastic potential energy, since the spring is unstrained initially. When the block comes to a momentary halt at the lowest point in its fall, the energy is all in the form of elastic potential energy. Since the block is again at rest, it again has no kinetic energy. The spring has been compressed, and gravitational potential energy has been converted entirely into elastic potential energy. The amount by which the spring is compressed is determined by the amount of gravitational potential energy that must be converted into elastic potential energy. The amount must be enough that the elastic potential energy equals the gravitational potential energy. Thus, we will use energy conservation to determine the magnitude of the spring's compression.

The compression of the spring is greater in the non-equilibrium case than in the equilibrium case. The reason is that in the non-equilibrium case, the block has been allowed to move, and its inertia carries it beyond its stationary equilibrium position on the spring. The compression of the spring must increase beyond that corresponding to the stationary equilibrium position in order to produce the force that is needed to decelerate the block to a momentary halt.

SOLUTION

a. As the block rests stationary on the spring, the downward-directed weight balances the upward-directed restoring force from the spring. The magnitude of the weight is mg, and the magnitude of the restoring force is given by Equation 10.2 without the minus sign as kx. Thus, we have

$$\underbrace{mg}_{\substack{\text{Magnitude of}\\\text{the weight}}} = \underbrace{kx}_{\substack{\text{Magnitude of}\\\text{the spring force}}} \quad \text{or} \quad x = \frac{mg}{k} = \frac{(0.64\text{ kg})(9.80\text{ m/s}^2)}{170\text{ N/m}} = \boxed{0.037\text{ m}}$$

b. The conservation of mechanical energy states that the final total mechanical energy E_f is equal to the initial total mechanical energy E_0. The expression for the total mechanical energy for an object on a spring is given by Equation 10.14, so that we have

$$\underbrace{\tfrac{1}{2}mv_f^2+\tfrac{1}{2}I\omega_f^2+mgh_f+\tfrac{1}{2}kx_f^2}_{E_f}=\underbrace{\tfrac{1}{2}mv_0^2+\tfrac{1}{2}I\omega_0^2+mgh_0+\tfrac{1}{2}kx_0^2}_{E_0}$$

The block does not rotate, so the angular speeds ω_f and ω_0 are zero. Since the block com… to a momentary halt on the spring and is released from rest, the translational speeds v_f and… are also zero. Because the spring is initially unstrained, the initial displacement x_0 of th… spring is likewise zero. Thus, the above expression can be simplified as follows:

$$mgh_f+\tfrac{1}{2}kx_f^2=mgh_0 \quad \text{or} \quad \tfrac{1}{2}kx_f^2=mg\left(h_0-h_f\right)$$

The term h_0-h_f is the amount by which the spring has compressed, or $h_0-h_f=x_f$. Maki… this substitution into the simplified energy-conservation equation gives

$$\tfrac{1}{2}kx_f^2=mg\left(h_0-h_f\right)=mgx_f \quad \text{or} \quad \tfrac{1}{2}kx_f=mg$$

Solving for x_f, we find

$$x=\frac{2mg}{k}=\frac{2(0.64\text{ kg})(9.80\text{ m/s}^2)}{170\text{ N/m}}=\boxed{0.074\text{ m}}$$

As expected, the spring compresses more in the non-equilibrium situation.

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37. SSM **_REASONING_** The angular frequency ω (in rad/s) is given by $\omega=\sqrt{\frac{k}{m}}$ (Equation 10.11), where k is the spring constant and m is the mass of the object. However we are given neither k nor m. Instead, we are given information about how much the spring is compressed and the launch speed of the object. Once launched, the object has kinetic energy, which is related to its speed. Before launching, the spring/object system has elastic potential energy, which is related to the amount by which the spring is compressed. This suggests that we apply the principle of conservation of mechanical energy in order to use the given information. This principle indicates that the total mechanical energy of the system is the same after the object is launched as it is before the launch. The resulting equation will provide us with the value of k/m that we need in order to determine the angular frequency from $\omega=\sqrt{\frac{k}{m}}$.

SOLUTION The conservation of mechanical energy states that the final total mechanical energy E_f is equal to the initial total mechanical energy E_0. The expression for the total mechanical energy for a spring/mass system is given by Equation 10.14, so that we have

$$\underbrace{\tfrac{1}{2}mv_f^2+\tfrac{1}{2}I\omega_f^2+mgh_f+\tfrac{1}{2}kx_f^2}_{E_f}=\underbrace{\tfrac{1}{2}mv_0^2+\tfrac{1}{2}I\omega_0^2+mgh_0+\tfrac{1}{2}kx_0^2}_{E_0}$$

Since the object does not rotate, the angular speeds ω_f and ω_0 are zero. Since the object is initially at rest, the initial translational speed v_0 is also zero. Moreover, the motion takes place horizontally, so that the final height h_f is the same as the initial height h_0. Lastly, the spring is unstrained after the launch, so that x_f is zero. Thus, the above expression can be simplified as follows:

$$\tfrac{1}{2}mv_f^2=\tfrac{1}{2}kx_0^2 \quad \text{or} \quad \frac{k}{m}=\frac{v_f^2}{x_0^2}$$

Substituting this result into Equation 10.11 shows that

$$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{v_f^2}{x_0^2}}=\frac{v_f}{x_0}=\frac{1.50\text{ m/s}}{0.0620\text{ m}}=\boxed{24.2\text{ rad/s}}$$

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38. **_REASONING_** As the sphere oscillates vertically, it is subject to conservative forces only: gravity and the elastic spring force. Therefore, its total mechanical energy $E=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2+mgh+\frac{1}{2}ky^2$ (Equation 10.14) is conserved, and we use the variable y to denote the vertical stretching of the spring relative to its unstrained length. We will apply the energy conservation principle to find the spring constant k.

SOLUTION In terms of the final and initial total mechanical energies E_f and E_0, the conservation principle gives us the following starting point:

$$\underbrace{\tfrac{1}{2}mv_f^2+\tfrac{1}{2}I\omega_f^2+mgh_f+\tfrac{1}{2}ky_f^2}_{E_f}=\underbrace{\tfrac{1}{2}mv_0^2+\tfrac{1}{2}I\omega_0^2+mgh_0+\tfrac{1}{2}ky_0^2}_{E_0} \qquad (1)$$

We note that the sphere does not rotate, so the angular speeds ω_0 and ω_f are zero. Equation (1) then becomes

$$\tfrac{1}{2}mv_f^2+mgh_f+\tfrac{1}{2}ky_f^2=\tfrac{1}{2}mv_0^2+mgh_0+\tfrac{1}{2}ky_0^2 \qquad (2)$$

Solving Equation (2) for the spring constant k, we obtain

$$\tfrac{1}{2}k\left(y_f^2-y_0^2\right)=\tfrac{1}{2}m\left(v_0^2-v_f^2\right)+mg\left(h_0-h_f\right) \quad \text{or} \quad k=\frac{m\left(v_0^2-v_f^2\right)+2mg\left(h_0-h_f\right)}{y_f^2-y_0^2} \qquad (3)$$

Solving for $\Delta L_{\text{Tension}}$, we find that

$$\Delta L_{\text{Tension}} = \left(\frac{Y_{\text{Compression}}}{Y_{\text{Tension}}}\right)\Delta L_{\text{Compression}} = \left(\frac{9.4\times10^{9}\ \text{N/m}^2}{1.6\times10^{10}\ \text{N/m}^2}\right)\left(2.7\times10^{-5}\ \text{m}\right) = \boxed{1.6\times10^{-5}\ \text{m}}$$

79. ***REASONING*** The applied force required to stretch the bow string is given by Equation 10.1 as $F_x^{\text{Applied}} = k\,x$, where k is the spring constant and x is the displacement of the string. Solving for the displacement gives $x = F_x^{\text{Applied}}/k$.

SOLUTION The displacement of the bow string is

$$x = \frac{F_x^{\text{Applied}}}{k} = \frac{+240\ \text{N}}{480\ \text{N/m}} = \boxed{+0.50\ \text{m}}$$

80. ***REASONING AND SOLUTION*** Equation 10.20 gives the desired result. Solving for $\Delta V/V_0$ and taking the value for the bulk modulus B of aluminum from Table 10.3, we obtain

$$\frac{\Delta V}{V_0} = -\frac{\Delta P}{B} = -\frac{-1.01\times10^{5}\ \text{Pa}}{7.1\times10^{10}\ \text{N/m}^2} = \boxed{1.4\times10^{-6}}$$

81. ***REASONING AND SOLUTION***
a. Now look at conservation of energy before and after the split

Before $\qquad \frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$

Solving for the amplitude A gives

$$A = v_{\max}\sqrt{\frac{m}{k}}$$

After $\qquad \frac{1}{2}\left(\frac{m}{2}\right)v'^2 = \frac{1}{2}\left(\frac{m}{2}\right)v_{\max}^2 = \frac{1}{2}kA'^2$

Solving for the amplitude A' gives

$$A' = v_{\max}\sqrt{\frac{m}{2k}}$$

Therefore, we find that

$$A' = \frac{A}{\sqrt{2}} = \frac{5.08\times10^{-2}\ \text{m}}{\sqrt{2}} = \boxed{3.59\times10^{-2}\ \text{m}}$$

Similarly, for the frequency, we can show that

$$f' = f\sqrt{2} = (3.00\ \text{Hz})\sqrt{2} = \boxed{4.24\ \text{Hz}}$$

b. If the block splits at one of the extreme positions, the amplitude of the SHM would not change, so it would remain as $\boxed{5.08\times10^{-2}\ \text{m}}$

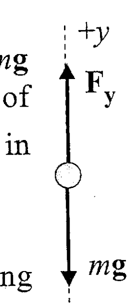
The frequency would be

$$f' = f\sqrt{2} = (3.00\ \text{Hz})\sqrt{2} = \boxed{4.24\ \text{Hz}}$$

82. ***REASONING***
a. The acceleration of the box is zero when the net force acting on it is zero, in accord with Newton's second law of motion. The net force includes the box's weight (directed downward) and the restoring force of the spring (directed upward). The condition that the net force is zero will allow us to determine the magnitude of the spring's displacement.

b. The speed of the box is zero when the spring is fully compressed, but the acceleration of the box is not zero at this instant. If the acceleration were zero, the box would be in equilibrium, and the net force on it would be zero. However, the box accelerates upward because the spring is exerting an upward force that is greater than the downward force due to the weight of the box. Thus, we cannot proceed as in part (a), so instead we will use energy conservation to determine the magnitude of the spring's displacement.

SOLUTION
a. The drawing at the right shows the two forces acting on the box: its weight mg and the restoring force $\mathbf{F}_y$ exerted by the spring. At the instant the acceleration of the box is zero, it is in equilibrium. According to Equation 4.9b, the net force ΣF_y in the y direction must be zero, $\Sigma F_y = 0$.



The restoring force is given by Equation 10.2 as $F_y = -ky$, where k is the spring constant and y is the displacement of the spring (assumed to be in the downward, or negative, direction). Thus, the condition for equilibrium can be written as

$$\underbrace{-ky + mg}_{\Sigma F_y} = 0 \quad \text{or} \quad y = -\frac{mg}{k}$$

Solving for the *magnitude* of the spring's displacement gives

$$\text{Magnitude of spring's displacement} = \frac{mg}{k} = \frac{(1.5\text{ kg})(9.80\text{ m/s}^2)}{450\text{ N/m}} = \boxed{3.3\times10^{-2}\text{ m}}$$

b. The conservation of mechanical energy states that the final total mechanical energy E_f is equal to the initial total mechanical energy E_0, or $E_f = E_0$ (Equation 6.9a). The expression for the total mechanical energy of an object is given by Equation 10.14. Thus, the conservation of total mechanical energy can be written as

$$\underbrace{\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f + \tfrac{1}{2}ky_f^2}_{E_f} = \underbrace{\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgh_0 + \tfrac{1}{2}ky_0^2}_{E_0}$$

We can simplify this equation by noting which variables are zero. Since the box comes to a momentary halt, $v_f = 0$ m/s. The box does not rotate, so its angular speed is zero, $\omega_f = \omega_0 =$ 0 rad/s. Initially, the spring is unstretched, so that $y_0 = 0$ m. Setting these terms equal to zero in the equation above gives

$$mgh_f + \tfrac{1}{2}ky_f^2 = \tfrac{1}{2}mv_0^2 + mgh_0$$

The vertical displacement $h_f - h_0$ through which the box falls is equal to the displacement y_f of the spring, so $y_f = h_f - h_0$. Note that y_f is negative, because h_f is less than h_0. The downward-moving box compresses the spring in the downward direction, which, as usual, we take to be the negative direction. Substituting this expression for y_f into the equation above and rearranging terms, we find that

$$\tfrac{1}{2}mv_0^2 - mg\underbrace{(h_f - h_0)}_{y_f} - \tfrac{1}{2}ky_f^2 = 0$$

or

$$\tfrac{1}{2}ky_f^2 + mgy_f - \tfrac{1}{2}mv_0^2 = 0$$

This is a quadratic equation in the variable y_f. The solution is

$$y_f = \frac{-mg \pm \sqrt{(mg)^2 - 4\left(\tfrac{1}{2}k\right)\left(-\tfrac{1}{2}mv_0^2\right)}}{2\left(\tfrac{1}{2}k\right)} = \frac{-mg}{k} \pm \sqrt{\left(\frac{mg}{k}\right)^2 + \left(\frac{m}{k}\right)v_0^2}$$

Substituting in the numbers, we find that

$$y_f = \frac{-(1.5\text{ kg})(9.80\text{ m/s}^2)}{450\text{ N/m}} \pm \sqrt{\left[\frac{(1.5\text{ kg})(9.80\text{ m/s}^2)}{450\text{ N/m}}\right]^2 + \left(\frac{1.5\text{ kg}}{450\text{ N/m}}\right)(0.49\text{ m/s})^2}$$

$$= 1.1\times10^{-2}\text{ m} \quad \text{or} \quad -7.6\times10^{-2}\text{ m}$$

The positive answer is discarded because the spring is compressed downward by the falling box, so the displacement of the spring is negative.

Therefore, the magnitude of the spring's displacement is $\boxed{7.6\times10^{-2}\text{ m}}$.

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83. ***REASONING***

a. The angular frequency ω (in rad/s) is given by Equation 10.11 $\left(\omega = \sqrt{\frac{k}{m}}\right)$, where k is the spring constant and m is the mass of the object. The frequency f (in Hz) can be obtained from the angular frequency by using Equation 10.6 ($\omega = 2pf$).

b. The block loses contact with the spring when the amplitude of the oscillation is sufficiently large. To understand why, consider the block at the very top of its oscillation cycle. There it is accelerating downward, with the maximum acceleration a_{max} of simple harmonic motion. Contact is maintained with the spring, as long as the magnitude of this acceleration is less than the magnitude g of the acceleration due to gravity. If a_{max} is greater than g, the end of the spring falls away from under the block. a_{max} is given by Equation 10.10 $\left(a_{max} = A\omega^2\right)$, from which we can obtain the amplitude A when $a_{max} = g$.

SOLUTION

a. From Equations 10.6 and 10.11 we have

$$\omega = 2\pi f = \sqrt{\frac{k}{m}} \quad \text{or} \quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{112\text{ N/m}}{0.400\text{ kg}}} = \boxed{2.66\text{ Hz}}$$

b. Using Equation 10.10 with $a_{max} = g$ gives

$$a_{max} = g = A\omega^2 \quad \text{or} \quad A = \frac{g}{\omega^2}$$

Substituting Equation 10.11 $\left(\omega = \sqrt{\frac{k}{m}}\right)$ into this result gives

$$A = \frac{g}{\omega^2} = \frac{g}{\left(\sqrt{k/m}\right)^2} = \frac{gm}{k} = \frac{(9.80\ \text{m/s}^2)(0.400\ \text{kg})}{112\ \text{N/m}} = \boxed{0.0350\ \text{m}}$$

84. ***REASONING*** As the climber falls, only two forces act on him: his weight and the elastic force of the nylon rope. Both of these forces are conservative forces, so the falling climber obeys the conservation of mechanical energy. We will use this conservation law to determine how much the rope is stretched when it breaks his fall and momentarily brings him to rest.

SOLUTION

The conservation of mechanical energy states that the final total mechanical energy E_f is equal to the initial total mechanical energy E_0, or $E_f = E_0$ (Equation 6.9a). The expression for the total mechanical energy of an object oscillating on a spring is given by Equation 10.14. Thus, the conservation of total mechanical energy can be written as

$$\underbrace{\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f + \tfrac{1}{2}ky_f^2}_{E_f} = \underbrace{\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgh_0 + \tfrac{1}{2}ky_0^2}_{E_0}$$

Before going any further, let's simplify this equation by noting which variables are zero. Since the climber starts and ends at rest, $v_f = v_0 = 0$ m/s. The climber does not rotate, so his angular speed is zero, $\omega_f = \omega_0 = 0$ rad/s. Initially, the spring is unstretched, so that $y_0 = 0$ m. Setting these terms to zero in the equation above gives

$$mgh_f + \tfrac{1}{2}ky_f^2 = mgh_0$$

Rearranging the terms in this equation, we have

$$\tfrac{1}{2}ky_f^2 + mg\underbrace{(h_f - h_0)}_{y_f - 0.750\ \text{m}} = 0$$

Note that the climber falls a distance of 0.750 m before the rope starts to stretch, and y_f is the displacement of the stretched rope. Since y_f points downward, it is considered to be negative. Thus, y_f – 0.750 m is the total downward displacement of the falling climber, which is also equal to $h_f - h_0$. With this substitution, the equation above becomes

$$\tfrac{1}{2}ky_f^2 + mgy_f - mg(0.750\ \text{m}) = 0$$

This is a quadratic equation in the variable y_f. Using the quadratic formula gives

$$y_f = \frac{-mg \pm \sqrt{(mg)^2 - 4\left(\tfrac{1}{2}k\right)\left[-mg(0.750\ \text{m})\right]}}{2\left(\tfrac{1}{2}k\right)}$$

$$y_f = \frac{-(86.0\ \text{kg})(9.80\ \text{m/s}^2)}{1.20\times10^3\ \text{N/m}}$$

$$\pm\frac{\sqrt{\left[(86.0\ \text{kg})(9.80\ \text{m/s}^2)\right]^2 - 4\left(\tfrac{1}{2}\right)(1.20\times10^3\ \text{N/m})\left[-(86.0\ \text{kg})(9.80\ \text{m/s}^2)(0.750\ \text{m})\right]}}{1.20\times10^3\ \text{N/m}}$$

There are two answers, y_f = +0.54 m and –1.95 m. Since the rope is stretched in the downward direction, which we have taken to be the negative direction, the displacement is –1.95 m. Thus, the amount that the rope is stretched is $\boxed{1.95\ \text{m}}$.

85. ***REASONING*** The two blocks and the spring between them constitute the system in this problem. Since the surface is frictionless and the weights of the blocks are balanced by the normal forces from the surface, no net external force acts on the system. Thus, the system's total mechanical energy and total linear momentum are each conserved. The conservation of these two quantities will give us two equations containing the two unknown speeds with which the blocks move away. Using these equations, we will be able to obtain the speeds.

SOLUTION The conservation of mechanical energy states that the final total mechanical energy E_f is equal to the initial total mechanical energy E_0. Only the translational kinetic energies of the blocks and the elastic potential energy of the spring are of interest here. There is no rotational kinetic energy since there is no rotation. Gravitational potential energy plays no role, because the surface is horizontal and the vertical height does not change. Thus, the expression for the conservation of the total mechanical energy is

$$\underbrace{\tfrac{1}{2}m_1v_{f1}^2 + \tfrac{1}{2}m_2v_{f2}^2 + \tfrac{1}{2}kx_f^2}_{E_f} = \underbrace{\tfrac{1}{2}m_1v_{01}^2 + \tfrac{1}{2}m_2v_{02}^2 + \tfrac{1}{2}kx_0^2}_{E_0}$$

Since the blocks are initially at rest, the initial translational speeds v_{01} and v_{02} are zero. In addition, the spring is neither compressed nor stretched after it is released, so that $x_f = 0$ m. Thus, the above expression can be simplified as follows:

$$\tfrac{1}{2}m_1v_{f1}^2 + \tfrac{1}{2}m_2v_{f2}^2 = \tfrac{1}{2}kx_0^2 \qquad (1)$$