17. ***REASONING*** According to $\Delta L = \alpha L_0 \Delta T$ (Equation 12.2), the factors that determine the amount ΔL by which the length of a rod changes are the coefficient of thermal expansion α, its initial length L_0, and the change ΔT in temperature. The materials from which the rods are made have different coefficients of thermal expansion (see Table 12.1 in the text). Also, the change in length is the same for each rod when the change in temperature is the same. Therefore, the initial lengths must be different to compensate for the fact that the expansion coefficients are different.

SOLUTION The change in length of the lead rod is (from Equation 12.2)

$$\Delta L_\mathrm{L} = \alpha_\mathrm{L} L_{0,\,\mathrm{L}} \Delta T \qquad (1)$$

Similarly, the change in length of the quartz rod is

$$\Delta L_\mathrm{Q} = \alpha_\mathrm{Q} L_{0,\,\mathrm{Q}} \Delta T \qquad (2)$$

In Equations (1) and (2) the temperature change ΔT is the same for both rods. Since each changes length by the same amount, $\Delta L_\mathrm{L} = \Delta L_\mathrm{Q}$. Equating Equations (1) and (2) and solving for $L_{0,\,\mathrm{Q}}$ yields

$$L_{0,\,\mathrm{Q}} = \left(\frac{\alpha_\mathrm{L}}{\alpha_\mathrm{Q}}\right) L_{0,\,\mathrm{L}} = \left[\frac{29\times10^{-6}\,(\mathrm{C}°)^{-1}}{0.50\times10^{-6}\,(\mathrm{C}°)^{-1}}\right](0.10\ \mathrm{m}) = \boxed{5.8\ \mathrm{m}}$$

The values for the coefficients of thermal expansion for lead and quartz have been taken from Table 12.1.

18. ***REASONING*** Each section of concrete expands as the temperature increases by an amount ΔT. The amount of the expansion ΔL is proportional to the initial length of the section, as indicated by Equation 12.2. Thus, to find the total expansion of the three sections, we can apply this expression to the total length of concrete, which is $L_0 = 3(2.4\ \mathrm{m})$. Since the two gaps in the drawing are identical, each must have a minimum width that is one half the total expansion.

SOLUTION Using Equation 12.2 and taking the value for the coefficient of thermal expansion for concrete from Table 12.1, we find

$$\Delta L = \alpha L_0 \Delta T = \left[12\times10^{-6}\,(\,\mathrm{C}°)^{-1}\right]\left[3(2.4\ \mathrm{m})\right](32\ \mathrm{C}°)$$

The minimum necessary gap width is one half this value or

$$\tfrac{1}{2}\left[12\times10^{-6}\,(\,\mathrm{C}°)^{-1}\right]\left[3(2.4\ \mathrm{m})\right](32\ \mathrm{C}°) = \boxed{1.4\times10^{-3}\ \mathrm{m}}$$

19. ***REASONING AND SOLUTION*** $\Delta L = \alpha L_0 \Delta T$ gives for the expansion of the aluminum

$$\Delta L_\mathrm{A} = \alpha_\mathrm{A} L_\mathrm{A} \Delta T \qquad (1)$$

and for the expansion of the brass

$$\Delta L_\mathrm{B} = \alpha_\mathrm{B} L_\mathrm{B} \Delta T \qquad (2)$$

The air gap will be closed when $\Delta L_\mathrm{A} + \Delta L_\mathrm{B} = 1.3\times10^{-3}\ \mathrm{m}$. Thus, taking the coefficients of thermal expansion for aluminum and brass from Table 12.1, adding Equations (1) and (2), and solving for ΔT, we find that

$$\Delta T = \frac{\Delta L_\mathrm{A} + \Delta L_\mathrm{B}}{\alpha_\mathrm{A} L_\mathrm{A} + \alpha_\mathrm{B} L_\mathrm{B}} = \frac{1.3\times10^{-3}\ \mathrm{m}}{\left[23\times10^{-6}\,(\mathrm{C}°)^{-1}\right](1.0\ \mathrm{m}) + \left[19\times10^{-6}\,(\mathrm{C}°)^{-1}\right](2.0\ \mathrm{m})} = 21\ \mathrm{C}°$$

The desired temperature is then

$$T = 28\ °\mathrm{C} + 21\ \mathrm{C}° = \boxed{49\ °\mathrm{C}}$$

20. ***REASONING*** The rod contracts upon cooling, and we can use Equation 12.2 to express the change ΔT in temperature as

$$\Delta T = \frac{\Delta L}{\alpha L_0} \qquad (1)$$

where ΔL and L_0 are the change in length and original length, respectively, and α is the coefficient of linear expansion for brass. Originally, the rod was stretched by the 860-N block that hangs from the lower end of the rod. This change in length can be found from Equation 10.17 as

$$\Delta L = \frac{F L_0}{Y A} \qquad (2)$$

where F is the magnitude of the stretching force, Y is Young's modulus for brass, and A is the cross-sectional area of the rod. Substituting Equation (2) into Equation (1) and noting that L_0 is algebraically eliminated from the final result, we have

$\Delta V = \beta V_0 \Delta T$, where β is the coefficient of volume expansion and V_0 is the initial volume of the block. Solving for ΔT gives

$$\Delta T = \frac{\Delta V}{\beta V_0}$$

Substituting this expression for ΔT into Equation 12.4 gives

$$Q = cm\Delta T = cm\left(\frac{\Delta V}{\beta V_0}\right)$$

SOLUTION The heat supplied to the block is

$$Q = cm\left(\frac{\Delta V}{\beta V_0}\right) = \frac{[750\ \text{J/(kg}\cdot\text{C}^\circ)](130\ \text{kg})(1.2\times10^{-5}\ \text{m}^3)}{[6.4\times10^{-5}\ (\text{C}^\circ)^{-1}](4.6\times10^{-2}\ \text{m}^3)} = \boxed{4.0\times10^5\ \text{J}}$$

---

56. ***REASONING*** Because the container is insulated, no heat is transferred to the surroundings. Therefore, in order to reach equilibrium at temperature T_{eq}, the oil must absorb an amount of heat Q_{oil} equal to the heat Q_{water} given up by the water. Neither the water nor the oil undergoes a phase change, so we will use $Q = cm\Delta T$ (Equation 12.4) to determine the amount of heat exchanged between the liquids. In Equation 12.4, c is the specific heat, m is the mass and ΔT is the temperature difference that each liquid undergoes. Thus, we have

$$Q_{oil} = c_{oil} m_{oil} \Delta T_{oil} \quad \text{and} \quad Q_{water} = c_{water} m_{water} \Delta T_{water} \tag{1}$$

For the water, the difference between the higher and lower temperatures is $\Delta T_{water} = 90.0\ ^\circ\text{C} - T_{eq}$. With this substitution, the expression for Q_{water} becomes

$$Q_{water} = c_{water} m_{water} (90.0\ ^\circ\text{C} - T_{eq}) \tag{2}$$

For the oil, the temperature change ΔT_{oil} is related to the increase ΔV in its volume by $\Delta V = \beta V_0 \Delta T_{oil}$ (Equation 12.3), where β is the coefficient of volume expansion of the oil, and V_0 is the volume of the oil before the water is added. Solving Equation 12.3 for ΔT_{oil}, we obtain

$$\Delta T_{oil} = \frac{\Delta V}{\beta V_0} \tag{3}$$

Substituting Equation (3) into the first of Equations (1) yields

$$Q_{oil} = \frac{c_{oil} m_{oil} \Delta V}{\beta V_0} \tag{4}$$

The mass m_{oil} of the oil in the container is related to its density ρ and volume V_0 according to $\rho = m_{oil}/V_0$ (Equation 11.1). Solving Equation 11.1 for the mass of the oil yields $m_{oil} = \rho V_0$, which we substitute into Equation (4):

$$Q_{oil} = \frac{c_{oil}\rho \cancel{V_0} \Delta V}{\beta \cancel{V_0}} = \frac{c_{oil}\rho\,\Delta V}{\beta} \tag{5}$$

SOLUTION Equation (5) gives the amount of heat absorbed by the oil, so it must be equal to Equation (2), which gives the amount of heat lost by the water. Therefore, we have

$$\frac{c_{oil}\rho\,\Delta V}{\beta} = c_{water} m_{water}(90.0\ ^\circ\text{C} - T_{eq}) \quad \text{or} \quad 90.0\ ^\circ\text{C} - T_{eq} = \frac{c_{oil}\rho\,\Delta V}{\beta c_{water} m_{water}} \tag{6}$$

Solving Equation (6) for the equilibrium temperature, we obtain

$$T_{eq} = 90.0\ ^\circ\text{C} - \frac{c_{oil}\rho\,\Delta V}{\beta c_{water} m_{water}}$$

$$= 90.0\ ^\circ\text{C} - \frac{[1970\ \text{J/(kg}\cdot\text{C}^\circ)](924\ \text{kg/m}^3)(1.20\times10^{-5}\ \text{m}^3)}{[721\times10^{-6}\ (\text{C}^\circ)^{-1}][4186\ \text{J/(kg}\cdot\text{C}^\circ)](0.125\ \text{kg})} = \boxed{32.1\ ^\circ\text{C}}$$

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57. $\boxed{\text{SSM}}$ ***REASONING*** Heat Q_1 must be added to raise the temperature of the aluminum in its solid phase from 130 °C to its melting point at 660 °C. According to Equation 12.4, $Q_1 = cm\Delta T$. The specific heat c of aluminum is given in Table 12.2. Once the solid aluminum is at its melting point, additional heat Q_2 must be supplied to change its phase from solid to liquid. The additional heat required to melt or liquefy the aluminum is $Q_2 = mL_f$, where L_f is the latent heat of fusion of aluminum. Therefore, the total amount of heat which must be added to the aluminum in its solid phase to liquefy it is

$$Q_{total} = Q_1 + Q_2 = m(c\Delta T + L_f)$$

SOLUTION Substituting values, we obtain

$$Q_{total} = (0.45\ \text{kg})\ \{[9.00\times10^2\ \text{J/(kg}\cdot\text{C}^\circ)](660\ ^\circ\text{C} - 130\ ^\circ\text{C}) + 4.0\times10^5\ \text{J/kg}\} = \boxed{3.9\times10^5\ \text{J}}$$

58. ***REASONING***

a. When water changes from the liquid to the ice phase at 0 °C, the amount of heat released is given by $Q = mL_f$ (Equation 12.5), where m is the mass of the water and L_f is its latent heat of fusion.

b. When heat Q is supplied to the tree, its temperature changes by an amount ΔT. The relation between Q and ΔT is given by Equation 12.4 as $Q = cm\Delta T$, where c is the specific heat capacity and m is the mass. This equation can be used to find the change in the tree's temperature.

SOLUTION

a. Taking the latent heat of fusion for water as $L_f = 33.5 \times 10^4$ J/kg (see Table 12.3), we find that the heat released by the water when it freezes is

$$Q = mL_f = (7.2\text{ kg})(33.5\times10^4\text{ J/kg}) = \boxed{2.4\times10^6\text{ J}}$$

b. Solving Equation 12.4 for the change in temperature, we have

$$\Delta T = \frac{Q}{cm} = \frac{2.4\times10^6\text{ J}}{[2.5\times10^3\text{ J/(kg}\cdot\text{C°)}](180\text{ kg})} = \boxed{5.3\text{ C°}}$$

59. ***REASONING AND SOLUTION***

a. The latent heat of vaporization L_v of water is given in Table 12.3. To change water at 100.0 °C to steam we have from Equation 12.5 that

$$Q = mL_v = (2.00\text{ kg})(22.6\times10^5\text{ J/kg}) = \boxed{4.52\times10^6\text{ J}}$$

b. For liquid water at 0.0 °C we must include the heat needed to raise the temperature to the boiling point. This heat is depends on the specific heat c of water, which is given in Table 12.2. Using Equations 12.4 and 12.5, we have $Q = (cm\Delta T)_{\text{water}} + mL_v$

$$Q = [4186\text{ J/(kg}\cdot\text{C°)}](2.00\text{ kg})(100.0\text{ C°}) + 4.52\times10^6\text{ J} = \boxed{5.36\times10^6\text{ J}}$$

REASONING

a. The amount of heat Q required to melt a mass m of a substance is directly proportional to the latent heat of fusion L_f, according to $Q = mL_f$ (Equation 12.5). Since each object has the same mass and more heat is required to melt B, the substance from which B is made has the larger latent heat of fusion. We will use Equation 12.5 to determine the latent heat.

b. The amount of heat required to melt a substance is also directly proportional to its mass, according to Equation 12.5. If the mass is doubled, the heat required to melt the substance also doubles.

SOLUTION

a. According to Equation 12.5, the latent heats of fusion for A and B are

$$L_{f,A} = \frac{Q_A}{m} = \frac{3.0\times10^4\text{ J}}{3.0\text{ kg}} = \boxed{1.0\times10^4\text{ J/kg}}$$

$$L_{f,B} = \frac{Q_B}{m} = \frac{9.0\times10^4\text{ J}}{3.0\text{ kg}} = \boxed{3.0\times10^4\text{ J/kg}}$$

b. The amount of heat required to melt object A when its mass is 6.0 kg is

$$Q = m_A L_{f,A} = (6.0\text{ kg})(1.0\times10^4\text{ J/kg}) = \boxed{6.0\times10^4\text{ J}} \qquad (12.5)$$

61. SSM ***REASONING*** From the conservation of energy, the heat lost by the mercury is equal to the heat gained by the water. As the mercury loses heat, its temperature decreases; as the water gains heat, its temperature rises to its boiling point. Any remaining heat gained by the water will then be used to vaporize the water.

According to Equation 12.4, the heat lost by the mercury is $Q_{\text{mercury}} = (cm\Delta T)_{\text{mercury}}$. The heat required to vaporize the water is, from Equation 12.5, $Q_{\text{vap}} = (m_{\text{vap}}L_v)_{\text{water}}$. Thus, the total amount of heat gained by the water is $Q_{\text{water}} = (cm\Delta T)_{\text{water}} + (m_{\text{vap}}L_v)_{\text{water}}$.

SOLUTION

$$Q_{\substack{\text{lost by}\\\text{mercury}}} = Q_{\substack{\text{gained by}\\\text{water}}}$$

$$(cm\Delta T)_{\text{mercury}} = (cm\Delta T)_{\text{water}} + (m_{\text{vap}}L_v)_{\text{water}}$$

where $\Delta T_{\text{mercury}} = (205\text{ °C} - 100.0\text{ °C})$ and $\Delta T_{\text{water}} = (100.0\text{ °C} - 80.0\text{ °C})$. The specific heats of mercury and water are given in Table 12.2, and the latent heat of vaporization of water is given in Table 12.3. Solving for the mass of the water that vaporizes gives

$$T = \frac{-m_{\text{ice}}L_f + \rho_{\text{coffee}}V_{\text{coffee}}c_{\text{coffee}}(85\ ^\circ\text{C})}{\rho_{\text{coffee}}V_{\text{coffee}}c_{\text{coffee}} + m_{\text{ice}}c_{\text{water}}}$$

$$= \frac{-(2)(11\times10^{-3}\ \text{kg})(3.35\times10^5\ \text{J/kg}) + (1.0\times10^3\ \text{kg/m}^3)(150\times10^{-6}\ \text{m}^3)[4186\ \text{J/(kg}\cdot\text{C}^\circ)](85\ ^\circ\text{C})}{(1.0\times10^3\ \text{kg/m}^3)(150\times10^{-6}\ \text{m}^3)[4186\ \text{J/(kg}\cdot\text{C}^\circ)] + (2)(11\times10^{-3}\ \text{kg})[4186\ \text{J/(kg}\cdot\text{C}^\circ)]}$$

$$= \boxed{64\ ^\circ\text{C}}$$

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66. ***REASONING*** The droplets of water in the air give up an amount of heat Q_w while cooling to the freezing point, and an amount Q_f while freezing into snow. The snow loses a further amount of heat Q_s as it cools from the freezing point to the air temperature. The total amount of heat transferred to the air as the lake water is transformed into a blanket of snow is the sum of the heat transferred during all three processes:

$$Q_{\text{total}} = Q_w + Q_f + Q_s \tag{1}$$

The heat lost by the cooling water droplets is found from $Q_w = c_w m\Delta T_w$ (Equation 12.4), where m is the total mass of water sprayed into the air in a minute, c_w is the specific heat capacity of water, and $\Delta T_w = 12.0$ C° is the difference between the temperature of the lake water (12.0 °C) and the freezing point (0 °C). For the freezing process, the heat loss is given by $Q_f = mL_f$ (Equation 12.5), where L_f is the latent heat of fusion of water (see Table 12.3 in the text). Lastly, the heat lost by the snow as it cools to the air temperature is given by $Q_s = c_s m\Delta T_s$ (Equation 12.4), where $\Delta T_s = 7.0$ C° is the higher initial temperature of the snow (0.0 °C, the freezing point) minus the lower final temperature (- 7.0 °C, air temperature), and c_s is the specific heat capacity of snow.

SOLUTION Substituting $Q_w = c_w m\Delta T_w$ (Equation 12.4), $Q_f = mL_f$ (Equation 12.5), and $Q_s = c_s m\Delta T_s$ (Equation 12.4) into Equation (1) yields

$$Q_{\text{total}} = Q_w + Q_f + Q_s = c_w m\Delta T_w + mL_f + c_s m\Delta T_s = m(c_w\Delta T_w + L_f + c_s\Delta T_s) \tag{2}$$

The snow maker pumps and sprays lake water at a rate of 130 kg per minute, so the mass m of the water turned to snow in one minute is m = 130 kg. Equation (2) then yields the total amount of heat transferred to the air each minute:

$$Q = (130\ \text{kg})\left\{[4186\ \text{J/(kg}\cdot\text{C}^\circ)](12.0\ \text{C}^\circ) + 33.5\times10^4\ \text{J/kg} + [2.00\times10^3\ \text{J/(kg}\cdot\text{C}^\circ)](7.0\ \text{C}^\circ)\right\}$$

$$= \boxed{5.2\times10^7\ \text{J}}$$

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67. $\boxed{\text{SSM}}$ ***REASONING*** According to the statement of the problem, the initial state of the system is comprised of the ice and the steam. From the principle of energy conservation, the heat lost by the steam equals the heat gained by the ice, or $Q_{\text{steam}} = Q_{\text{ice}}$. When the ice and the steam are brought together, the steam immediately begins losing heat to the ice. An amount $Q_{1(\text{lost})}$ is released as the temperature of the steam drops from 130 °C to 100 °C, the boiling point of water. Then an amount of heat $Q_{2(\text{lost})}$ is released as the steam condenses into liquid water at 100 °C. The remainder of the heat lost by the "steam" $Q_{3(\text{lost})}$ is the heat that is released as the water at 100 °C cools to the equilibrium temperature of $T_{\text{eq}} = 50.0$ °C. According to Equation 12.4, $Q_{1(\text{lost})}$ and $Q_{3(\text{lost})}$ are given by

$$Q_{1(\text{lost})} = c_{\text{steam}}m_{\text{steam}}(T_{\text{steam}} - 100.0\ ^\circ\text{C}) \quad \text{and} \quad Q_{3(\text{lost})} = c_{\text{water}}m_{\text{steam}}(100.0\ ^\circ\text{C} - T_{\text{eq}})$$

$Q_{2(\text{lost})}$ is given by $Q_{2(\text{lost})} = m_{\text{steam}}L_v$, where L_v is the latent heat of vaporization of water. The total heat lost by the steam has three effects on the ice. First, a portion of this heat $Q_{1(\text{gained})}$ is used to raise the temperature of the ice to its melting point at 0.00 °C. Then, an amount of heat $Q_{2(\text{gained})}$ is used to melt the ice completely (we know this because the problem states that after thermal equilibrium is reached the liquid phase is present at 50.0 °C). The remainder of the heat $Q_{3(\text{gained})}$ gained by the "ice" is used to raise the temperature of the resulting liquid at 0.0 °C to the final equilibrium temperature. According to Equation 12.4, $Q_{1(\text{gained})}$ and $Q_{3(\text{gained})}$ are given by

$$Q_{1(\text{gained})} = c_{\text{ice}}m_{\text{ice}}(0.00\ ^\circ\text{C} - T_{\text{ice}}) \quad \text{and} \quad Q_{3(\text{gained})} = c_{\text{water}}m_{\text{ice}}(T_{\text{eq}} - 0.00\ ^\circ\text{C})$$

$Q_{2(\text{gained})}$ is given by $Q_{2(\text{gained})} = m_{\text{ice}}L_f$, where L_f is the latent heat of fusion of ice.

SOLUTION According to the principle of energy conservation, we have

$$Q_{\text{steam}} = Q_{\text{ice}}$$

$$Q_{1(\text{lost})} + Q_{2(\text{lost})} + Q_{3(\text{lost})} = Q_{1(\text{gained})} + Q_{2(\text{gained})} + Q_{3(\text{gained})}$$

or

$$c_{steam}m_{steam}(T_{steam} - 100.0\ °C) + m_{steam}L_v + c_{water}m_{steam}(100.0\ °C - T_{eq})$$

$$= c_{ice}m_{ice}(0.00\ °C - T_{ice}) + m_{ice}L_f + c_{water}m_{ice}(T_{eq} - 0.00\ °C)$$

Values for specific heats are given in Table 12.2, and values for the latent heats are given in Table 12.3. Solving for the ratio of the masses gives

$$\frac{m_{steam}}{m_{ice}} = \frac{c_{ice}(0.00\ °C - T_{ice}) + L_f + c_{water}(T_{eq} - 0.00\ °C)}{c_{steam}(T_{steam} - 100.0\ °C) + L_v + c_{water}(100.0\ °C - T_{eq})}$$

$$= \frac{\left[2.00\times10^3\ J/(kg\cdot C°)\right]\left[0.0\ °C-(-10.0°C)\right]+33.5\times10^4\ J/kg+\left[4186\ J/(kg\cdot C°)\right](50.0\ °C-0.0\ °C)}{\left[2020\ J/(kg\cdot C°)\right](130\ °C-100.0\ °C)+22.6\times10^5\ J/kg+\left[4186\ J/(kg\cdot C°)\right](100.0\ °C-50.0\ °C)}$$

or

$$\frac{m_{steam}}{m_{ice}} = \boxed{0.223}$$

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68. ***REASONING*** When the minimum mass m_w of water is used to solidify the molten gold, the heat Q_g lost by the gold is exactly the amount needed to raise the temperature of the water to the boiling point and then convert the water into steam. If the mass of the water is smaller than m_w, all of the water will be converted to steam before the gold has entirely solidified. The water first gains an amount of heat $Q_1 = c_w m_w \Delta T$ (Equation 12.4) when it is raised to the boiling point (100.0 °C), where $c_w = 4186\ J/(kg\cdot C°)$ is the specific heat capacity of water (see Table 12.2) and ΔT is the change in its temperature. When the boiling water is converted to steam, it absorbs a further amount of heat $Q_2 = m_w L_w$ (Equation 12.5), where $L_w = 22.6\times10^5$ J/kg is the latent heat of vaporization of water (see Table 12.3). The total amount of heat given off by the molten gold as it solidifies is found from $Q_g = m_g L_g$ (Equation 12.5), where m_g is the mass of the gold and $L_g = 6.28\times10^4$ J/kg is the latent heat of fusion of gold (see Table 12.3). The total amount of heat Q_w gained by the water is equal to the heat lost by the gold, so we have that

$$Q_w = Q_1 + Q_2 = Q_g \qquad (1)$$

SOLUTION Substituting $Q_1 = c_w m_w \Delta T$ (Equation 12.4), $Q_2 = m_w L_w$ (Equation 12.5), and $Q_g = m_g L_g$ (Equation 12.5) into Equation (1), we obtain

$$\underbrace{c_w m_w \Delta T + m_w L_w}_{\text{Heat gained by water}} = \underbrace{m_g L_g}_{\text{Heat lost by gold}} \qquad (2)$$

Solving Equation (2) for m_w yields

$$m_w\left(c_w \Delta T + L_w\right) = m_g L_g \qquad \text{or} \qquad m_w = \frac{m_g L_g}{c_w \Delta T + L_w}$$

Therefore,

$$m_w = \frac{(0.180\ kg)\left(6.28\times10^4\ J/kg\right)}{\left[4186\ J/(kg\cdot C°)\right](100.0\ °C-23.0\ °C)+22.6\times10^5\ J/kg} = \boxed{4.38\times10^{-3}\ kg}$$

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69. SSM WWW ***REASONING*** The system is comprised of the unknown material, the glycerin, and the aluminum calorimeter. From the principle of energy conservation, the heat gained by the unknown material is equal to the heat lost by the glycerin and the calorimeter. The heat gained by the unknown material is used to melt the material and then raise its temperature from the initial value of −25.0 °C to the final equilibrium temperature of $T_{eq} = 20.0$ °C.

SOLUTION

$$Q_{\substack{\text{gained by}\\ \text{unknown}}} = Q_{\substack{\text{lost by}\\ \text{glycerine}}} + Q_{\substack{\text{lost by}\\ \text{calorimeter}}}$$

$$m_u L_f + c_u m_u \Delta T_u = c_{gl} m_{gl} \Delta T_{gl} + c_{al} m_{al} \Delta T_{al}$$

Taking values for the specific heat capacities of glycerin and aluminum from Table 12.2, we have

$$(0.10\ kg)L_f + [160\ J/(kg\cdot C°)](0.10\ kg)(45.0\ C°) = [2410\ J/(kg\cdot C°)](0.100\ kg)(7.0\ C°)$$
$$+ [9.0\times10^2\ J/(kg\cdot C°)](0.150\ kg)(7.0\ C°)$$

Solving for L_f yields,

$$L_f = \boxed{1.9\times10^4\ J/kg}$$

95. ***REASONING*** Since all of the heat generated by friction goes into the block of ice, only this heat provides the heat needed to melt some of the ice. Since the surface on which the block slides is horizontal, the gravitational potential energy does not change, and energy conservation dictates that the heat generated by friction equals the amount by which the kinetic energy decreases or $Q_{\text{Friction}} = \frac{1}{2}Mv_0^2 - \frac{1}{2}Mv^2$, where v_0 and v are, respectively, the initial and final speeds and M is the mass of the block. In reality, the mass of the block decreases as the melting proceeds. However, only a very small amount of ice melts, so we may consider M to be essentially constant at its initial value. The heat Q needed to melt a mass m of water is given by Equation 12.5 as $Q = mL_f$, where L_f is the latent heat of fusion. Thus, by equating Q_{Friction} to mL_f and solving for m, we can determine the mass of ice that melts.

SOLUTION Equating Q_{Friction} to mL_f and solving for m gives

$$Q_{\text{Friction}} = \tfrac{1}{2}Mv_0^2 - \tfrac{1}{2}Mv^2 = mL_f$$

$$m = \frac{M\left(v_0^2 - v^2\right)}{2L_f} = \frac{(42\text{ kg})\left[(7.3\text{ m/s})^2 - (3.5\text{ m/s})^2\right]}{2\left(33.5\times10^4\text{ J/kg}\right)} = \boxed{2.6\times10^{-3}\text{ kg}}$$

We have taken the value for the latent heat of fusion for water from Table 12.3.

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96. ***REASONING*** To freeze either liquid, heat must be removed to cool the liquid to its freezing point. In either case, the heat Q that must be removed to lower the temperature of a substance of mass m by an amount ΔT is given by Equation 12.4 as $Q = cm\Delta T$, where c is the specific heat capacity. The amount ΔT by which the temperature is lowered is the initial temperature T_0 minus the freezing point temperature T. Once the liquid has been cooled to its freezing point, additional heat must be removed to convert the liquid into a solid at the freezing point. The heat Q that must be removed to freeze a mass m of liquid into a solid is given by Equation 12.5 as $Q = mL_f$, where L_f is the latent heat of fusion. The total heat to be removed, then, is the sum of that specified by Equation 12.4 and that specified by Equation 12.5, or $Q_{\text{Total}} = cm\,(T_0 - T) + mL_f$. Since we know that same amount of heat is removed from each liquid, we can set Q_{Total} for liquid A equal to Q_{Total} for liquid B and solve the resulting equation for $L_{f,\,A} - L_{f,\,B}$.

SOLUTION Setting Q_{Total} for liquid A equal to Q_{Total} for liquid B gives

$$c_A m\,(T_0 - T_A) + mL_{f,\,A} = c_B m\,(T_0 - T_B) + mL_{f,\,B}$$

Noting that the mass m can be eliminated algebraically from this result and solving for $L_{f,\,A} - L_{f,\,B}$, we find

$$L_{f,\,A} - L_{f,\,B} = c_B\left(T_0 - T_B\right) - c_A\left(T_0 - T_A\right)$$

$$= \left[2670\text{ J/(kg}\cdot\text{C}^\circ)\right]\left[25.0\,^\circ\text{C} - (-96.0\,^\circ\text{C})\right]$$

$$- \left[1850\text{ J/(kg}\cdot\text{C}^\circ)\right]\left[25.0\,^\circ\text{C} - (-68.0\,^\circ\text{C})\right] = \boxed{1.51\times10^5\text{ J/kg}}$$

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97. ***REASONING AND SOLUTION*** We wish to convert 2.0% of the heat Q into gravitational potential energy, i.e., $(0.020)Q = mgh$. Thus,

$$mg = \frac{(0.020)Q}{h} = \frac{(0.020)\left(110\ \cancel{\text{Calories}}\right)\left(\dfrac{4186\text{ J}}{1\ \cancel{\text{Calorie}}}\right)}{2.1\text{ m}} = \boxed{4.4\times10^3\text{ N}}$$

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98. ***REASONING*** Young's modulus Y can be obtained from $F = Y\left(\Delta L/L_0\right)A$ (Equation 10.17), where F is the magnitude of the stretching force applied to the ruler, ΔL and L_0 are the change in length and original length, respectively, and A is the cross-sectional area. Solving for Y gives

$$Y = \frac{F L_0}{A\,\Delta L}$$

The change in the length of the ruler is given by $\Delta L = \alpha L_0 \Delta T$ (Equation 12.2), where α is the coefficient of linear expansion and ΔT is the amount by which the temperature changes. Substituting this expression for ΔL into the equation above for Y gives the desired result.

SOLUTION Substituting $\Delta L = \alpha L_0 \Delta T$ into $Y = F L_0/(A\,\Delta L)$ and using the fact that $\Delta T = 39\text{ C}^\circ$, we find that

$$Y = \frac{F L_0}{A\,\Delta L} = \frac{F\,\cancel{L_0}}{A\left(\alpha\,\cancel{L_0}\,\Delta T\right)}$$

$$= \frac{F}{A\,\alpha\,\Delta T} = \frac{1.2\times10^3\text{ N}}{\left(1.6\times10^{-5}\text{ m}^2\right)\left[2.5\times10^{-5}\,(\text{C}^\circ)^{-1}\right](39\text{ C}^\circ)} = \boxed{7.7\times10^{10}\text{ N/m}^2}$$
