7. $\boxed{\text{SSM}}$ $\boxed{\text{WWW}}$ ***REASONING AND SOLUTION*** Values for the thermal conductivities of Styrofoam and air are given in Table 11.1. The conductance of an 0.080 mm thick sample of Styrofoam of cross-sectional area A is

$$\frac{k_s A}{L_s} = \frac{[0.010\ \text{J/(s}\cdot\text{m}\cdot\text{C°)}]\ A}{0.080\times 10^{-3}\ \text{m}} = [125\ \text{J/(s}\cdot\text{m}^2\cdot\text{C°)}]\ A$$

The conductance of a 3.5 mm thick sample of air of cross-sectional area A is

$$\frac{k_a A}{L_a} = \frac{[0.0256\ \text{J/(s}\cdot\text{m}\cdot\text{C°)}]\ A}{3.5\times 10^{-3}\ \text{m}} = [7.3\ \text{J/(s}\cdot\text{m}^2\cdot\text{C°)}]\ A$$

Dividing the conductance of Styrofoam by the conductance of air for samples of the same cross-sectional area A, gives

$$\frac{[125\ \text{J/(s}\cdot\text{m}^2\cdot\text{C°)}]\ A}{[7.3\ \text{J/(s}\cdot\text{m}^2\cdot\text{C°)}]\ A} = 17$$

Therefore, the body can adjust the conductance of the tissues beneath the skin by $\boxed{\text{a factor of 17}}$.

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8. ***REASONING*** The inner radius r_{in} and outer radius r_{out} of the pipe determine the cross-sectional area A (copper only) of the pipe, which is the difference between the area A_{out} of a circle with radius r_{out} and the area A_{in} of a circle with a radius r_{in}:

$$A = A_{out} - A_{in} = \pi r_{out}^2 - \pi r_{in}^2 \tag{1}$$

The heat Q that flows along the pipe in a time $t = 15$ min is related to the cross-sectional area A by $Q = \frac{(kA\,\Delta T)t}{L}$ (Equation 13.1), where k is the thermal conductivity of copper (see Table 13.1 in the text), L is the length of the pipe, and ΔT is the temperature difference between the faucet, where the temperature is 4.0 °C, and the point on the pipe 3.0 m from the faucet where the temperature is 25 °C: $\Delta T = 25\ °\text{C} - 4.0\ °\text{C} = 21\ \text{C°}$. We will use Equation (1) and Equation 13.1 to find the inner radius of the pipe.

SOLUTION Solving Equation (1) for the inner radius r_{in} of the pipe yields

$$\pi r_{in}^2 = \pi r_{out}^2 - A \quad \text{or} \quad r_{in}^2 = r_{out}^2 - \frac{A}{\pi} \quad \text{or} \quad r_{in} = \sqrt{r_{out}^2 - \frac{A}{\pi}} \tag{2}$$

An expression for the cross-sectional area A of the pipe may be obtained by solving $Q = \frac{(kA\,\Delta T)t}{L}$ (Equation 13.1):

$$A = \frac{QL}{(k\Delta T)t} \tag{3}$$

Substituting Equation (3) into Equation (2) yields

$$r_{in} = \sqrt{r_{out}^2 - \frac{QL}{\pi(k\Delta T)t}} \tag{4}$$

Before using Equation (4), we convert the time t from minutes to seconds:

$$t = (15\ \cancel{\text{min}})\left(\frac{60\ \text{s}}{1\ \cancel{\text{min}}}\right) = 9.0\times 10^2\ \text{s}$$

The inner radius of the pipe is, therefore,

$$r_{in} = \sqrt{(0.013\ \text{m})^2 - \frac{(270\ \text{J})(3.0\ \text{m})}{\pi\left[390\ \text{J/(s}\cdot\text{m}\cdot\text{C°)}\right](25\ °\text{C} - 4.0\ °\text{C})(9.0\times 10^2\ \text{s})}} = \boxed{0.012\ \text{m}}$$

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9. ***REASONING*** The heat Q conducted along the bar is given by the relation $Q = \frac{(kA\Delta T)t}{L}$ (Equation 13.1). We can determine the temperature difference between the hot end of the bar and a point 0.15 m from that end by solving this equation for ΔT and noting that the heat conducted per second is Q/t and that $L = 0.15$ m.

SOLUTION Solving Equation 13.1 for the temperature difference, using the fact that $Q/t = 3.6$ J/s, and taking the thermal conductivity of brass from Table 13.1, yield

$$\Delta T = \frac{(Q/t)L}{kA} = \frac{(3.6\ \text{J/s})(0.15\ \text{m})}{\left[110\ \text{J/(s}\cdot\text{m}\cdot\text{C°)}\right](2.6\times 10^{-4}\ \text{m}^2)} = 19\ \text{C°}$$

The temperature at a distance of 0.15 m from the hot end of the bar is

$$T = 306\ °\text{C} - 19\ \text{C°} = \boxed{287\ °\text{C}}$$

$$Q_{\text{Total}} = Q_{\text{Steel}} + Q_{\text{Iron}}$$

$$= \left[\frac{(kA\Delta T)t}{L}\right]_{\text{Steel}} + \left[\frac{(kA\Delta T)t}{L}\right]_{\text{Iron}} = \left[k_{\text{Steel}}L^2 + k_{\text{Iron}}\left(\frac{\pi}{2}-1\right)L^2\right]\frac{(\Delta T)t}{L}$$

$$= \left[\left(14\ \frac{\text{J}}{\text{s}\cdot\text{m}\cdot\text{C}^\circ}\right)(0.010\ \text{m})^2 + \left(79\ \frac{\text{J}}{\text{s}\cdot\text{m}\cdot\text{C}^\circ}\right)\left(\frac{\pi}{2}-1\right)(0.010\ \text{m})^2\right]$$

$$\times\frac{(78\ ^\circ\text{C} - 18\ ^\circ\text{C})(120\ \text{s})}{0.50\ \text{m}} = \boxed{85\ \text{J}}$$

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12. ***REASONING***

a. The heat Q conducted through the tile in a time t is given by $Q = \frac{(kA\,\Delta T)t}{L}$ (Equation 13.1), where k is the thermal conductivity of the tile, A is its cross-sectional area, L is the distance between the outer and inner surfaces, and ΔT is the temperature difference between the outer and inner surfaces.

b. We will use $Q = cm\Delta T$ (Equation 12.4) to find the increase ΔT in the temperature of a mass $m = 2.0$ kg of water when an amount of heat Q is transferred to it.

SOLUTION

a. The time $t = 5.0$ min must be converted to SI units (seconds):

$$t = (5.0\ \cancel{\text{min}})\left(\frac{60\ \text{s}}{1\ \cancel{\text{min}}}\right) = 3.0\times10^2\ \text{s}$$

Because the tile is cubical, its thickness is equal to the length L of one of its sides, and its cross-sectional area A is the product of two of its side lengths $A = (L)(L) = L^2$. Applying $Q = \frac{(kA\,\Delta T)t}{L}$ (Equation 13.1), we obtain the amount of heat conducted by the tile in five minutes:

$$Q = \frac{(kA\,\Delta T)t}{L} = \frac{\left(kL^{\cancel{2}}\,\Delta T\right)t}{\cancel{L}} = (kL\,\Delta T)t$$

$$= \left[0.065\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}^\circ)\right](0.10\ \text{m})(1150\ ^\circ\text{C} - 20.0\ ^\circ\text{C})(3.0\times10^2\ \text{s}) = \boxed{2200\ \text{J}}$$

b. Solving $Q = cm\Delta T$ (Equation 12.4) for the increase ΔT in temperature, we obtain

$$\Delta T = \frac{Q}{cm} \quad (1)$$

In Equation (1), we will use the value of Q found in part (a), and the specific heat capacity c of water given in Table 12.2 in the text. With these values, the increase in temperature of two liters of water is

$$\Delta T = \frac{2200\ \text{J}}{\left[4186\ \text{J}/(\text{kg}\cdot\text{C}^\circ)\right](2.0\ \text{kg})} = \boxed{0.26\ \text{C}^\circ}$$

---

13. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The rate of heat transfer is the same for all three materials so

$$Q/t = k_p A\Delta T_p/L = k_b A\Delta T_b/L = k_w A\Delta T_w/L$$

Let T_i be the inside temperature, T_1 be the temperature at the plasterboard-brick interface, T_2 be the temperature at the brick-wood interface, and T_o be the outside temperature. Then

$$k_pT_i - k_pT_1 = k_bT_1 - k_bT_2 \quad (1)$$

and

$$k_bT_1 - k_bT_2 = k_wT_2 - k_wT_o \quad (2)$$

Solving (1) for T_2 gives

$$T_2 = (k_p + k_b)T_1/k_b - (k_p/k_b)T_i$$

a. Substituting this into (2) and solving for T_1 yields

$$T_1 = \frac{(k_p/k_b)(1 + k_w/k_b)T_i + (k_w/k_b)T_0}{(1 + k_w/k_b)(1 + k_p/k_b) - 1} = \boxed{21\ ^\circ\text{C}}$$

b. Using this value in (1) yields

$$\boxed{T_2 = 18\ ^\circ\text{C}}$$

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14. ***REASONING*** If m kilograms of ice melt in t seconds, then $Q = mL_f$ (Equation 12.5) joules of heat must be delivered to the ice through the copper rod in t seconds, where $L_f = 33.5\times10^4$ J/kg is the latent heat of fusion of water. The mass of ice per second

where P_{bulb} is the power rating of the light bulb. Therefore,

$$t_{blackbody} = \frac{(100.0\text{ J/s})\,(3600\text{ s})}{\left[5.67\times10^{-8}\text{ J/(s}\cdot\text{m}^2\cdot\text{K}^4)\right](303\text{ K})^4\left[(6\text{ sides})(0.0100\text{ m})^2/\text{side}\right]}$$

$$\times\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1\text{ d}}{24\text{ h}}\right) = \boxed{14.5\text{ d}}$$

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22. ***REASONING*** According to the Stefan-Boltzmann law, the radiant power emitted by the "radiator" is $\frac{Q}{t} = e\sigma T^4 A$ (Equation 13.2), where Q is the energy radiated in a time t, e is the emissivity of the surface, σ is the Stefan-Boltzmann constant, T is the temperature in Kelvins, and A is the area of the surface from which the radiant energy is emitted. We will apply this law to the "radiator" before and after it is painted. In either case, the same radiant power is emitted.

SOLUTION Applying the Stefan-Boltzmann law, we obtain the following:

$$\left(\frac{Q}{t}\right)_{after} = e_{after}\,\sigma T_{after}^4 A \quad\text{and}\quad \left(\frac{Q}{t}\right)_{before} = e_{before}\,\sigma T_{before}^4 A$$

Since the same radiant power is emitted before and after the "radiator" is painted, we have

$$\left(\frac{Q}{t}\right)_{after} = \left(\frac{Q}{t}\right)_{before} \quad\text{or}\quad e_{after}\,\sigma T_{after}^4 A = e_{before}\,\sigma T_{before}^4 A$$

The terms σ and A can be eliminated algebraically, so this result becomes

$$e_{after}\,\cancel{\sigma} T_{after}^4 \cancel{A} = e_{before}\,\cancel{\sigma} T_{before}^4 \cancel{A} \quad\text{or}\quad e_{after}\,T_{after}^4 = e_{before}\,T_{before}^4$$

Remembering that the temperature in the Stefan-Boltzmann law must be expressed in Kelvins, so that $T_{before} = 62\text{ °C} + 273 = 335\text{ K}$ (see Section 12.2), we find that

$$T_{after}^4 = \frac{e_{before}\,T_{before}^4}{e_{after}} \quad\text{or}\quad T_{after} = \sqrt[4]{\frac{e_{before}}{e_{after}}}\,(T_{before}) = \sqrt[4]{\frac{0.75}{0.50}}\,(335\text{ K}) = 371\text{ K}$$

On the Celsius scale, this temperature is $371\text{ K} - 273 = \boxed{98\text{ °C}}$.

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23. ***REASONING*** The radiant energy Q absorbed by the person's head is given by $Q = e\sigma T^4 A t$ (Equation 13.2), where e is the emissivity, σ is the Stefan-Boltzmann constant, T is the Kelvin temperature of the environment surrounding the person ($T = 28\text{ °C} + 273 = 301\text{ K}$), A is the area of the head that is absorbing the energy, and t is the time. The radiant energy absorbed per second is $Q/t = e\sigma T^4 A$.

SOLUTION

a. The radiant energy absorbed per second by the person's head when it is covered with hair ($e = 0.85$) is

$$\frac{Q}{t} = e\sigma T^4 A = (0.85)\left[5.67\times10^{-8}\text{ J/}\left(\text{s}\cdot\text{m}^2\cdot\text{K}^4\right)\right](301\text{ K})^4\left(160\times10^{-4}\text{ m}^2\right) = \boxed{6.3\text{ J/s}}$$

b. The radiant energy absorbed per second by a bald person's head ($e = 0.65$) is

$$\frac{Q}{t} = e\sigma T^4 A = (0.65)\left[5.67\times10^{-8}\text{ J/}\left(\text{s}\cdot\text{m}^2\cdot\text{K}^4\right)\right](301\text{ K})^4\left(160\times10^{-4}\text{ m}^2\right) = \boxed{4.8\text{ J/s}}$$

---

24. ***REASONING*** The net radiant power of the baking dish is given by $P_{net} = e\sigma A\left(T^4 - T_0^4\right)$, (Equation 13.3), where e is the emissivity of the dish, σ is the Stefan-Boltzmann constant, A is the total surface area of the dish, T is the Kelvin temperature of the dish, and T_0 is the temperature of the kitchen. As the dish cools down, both its net radiant power P_{net} and temperature T decrease, but its emissivity e and surface area A remain constant. We will therefore solve Equation 13.3 for the product of these constant quantities and the Stefan-Boltzmann constant:

$$e\sigma A = \frac{P_{net}}{\left(T^4 - T_0^4\right)} \qquad (1)$$

Because the left side of Equation (1) is constant, the ratio on the right hand side cannot change as the temperature T decreases. We will use this fact to determine the radiant power $P_{net,1}$ of the baking dish when its temperature is 175 °C.

SOLUTION From Equation (1), we have that

$$\frac{P_{net,1}}{\left(T_1^4 - T_0^4\right)} = \frac{P_{net,2}}{\left(T_2^4 - T_0^4\right)} \quad\text{or}\quad P_{net,1} = P_{net,2}\left[\frac{T_1^4 - T_0^4}{T_2^4 - T_0^4}\right] \qquad (2)$$

In Equation (2), $P_{net,2} = 12.0\text{ W}$ is the net radiant power when the temperature is 35 °C. We first convert temperatures to the Kelvin scale by adding 273 to each temperature given in degrees Celsius (see Equation 12.1). The initial temperature of the baking dish is

$T_1 = 175\ °C + 273 = 448\ K$, its final temperature is $T_2 = 35\ °C + 273 = 308\ K$, and the room temperature is $T_0 = 22\ °C + 273 = 295\ K$. Equation (2), then, gives the net radiant power when the dish is first brought out of the oven:

$$P_{net,1} = (12.0\ W)\left[\frac{(448\ K)^4 - (295\ K)^4}{(308\ K)^4 - (295\ K)^4}\right] = \boxed{275\ W}$$

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25. ***REASONING*** According to the discussion in Section 13.3, the net power P_{net} radiated by the person is $P_{net} = e\sigma A\left(T^4 - T_0^4\right)$, where e is the emissivity, σ is the Stefan-Boltzmann constant, A is the surface area, and T and T_0 are the temperatures of the person and the environment, respectively. Since power is the change in energy per unit time (see Equation 6.10b), the time t required for the person to emit the energy Q contained in the dessert is $t = Q/P_{net}$.

SOLUTION The time required to emit the energy from the dessert is

$$t = \frac{Q}{P_{net}} = \frac{Q}{e\sigma A\left(T^4 - T_0^4\right)}$$

The energy is $Q = (260\ \text{Calories})\left(\frac{4186\ J}{1\ \text{Calorie}}\right)$, and the Kelvin temperatures are $T = 36\ °C + 273 = 309\ K$ and $T_0 = 21\ °C + 273 = 294\ K$. The time is

$$t = \frac{(260\ \text{Calories})\left(\frac{4186\ J}{1\ \text{Calorie}}\right)}{(0.75)\left[5.67\times10^{-8}\ J/\left(s\cdot m^2\cdot K^4\right)\right]\left(1.3\ m^2\right)\left[(309\ K)^4 - (294\ K)^4\right]} = \boxed{1.2\times10^4\ s}$$

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26. ***REASONING*** The power radiated by an object is given by $Q/t = e\sigma T^4 A$ (Equation 13.2), where e is the emissivity of the object, σ is the Stefan-Boltzmann constant, T is the temperature (in kelvins) of the object, and A is its surface area.

The power that the object absorbs from the room is given by $Q/t = e\sigma T_0^{\,4} A$. Except for the temperature T_0 of the room, this expression has the same form as that for the power radiated by the object. Note especially that the area A is the surface area of the ***object, not the room***. Review Example 8 in the text to understand this important point.

SOLUTION The object emits three times as much power as it absorbs from the room, so it follows that $(Q/t)_{emitted} = 3(Q/t)_{absorbed}$. Using the Stefan-Boltzmann law for each of the powers, we find

$$\underbrace{e\sigma T^4 A}_{\text{Power emitted}} = \underbrace{3e\sigma T_0^4 A}_{\text{Power absorbed}}$$

Solving for the temperature T of the object gives

$$T = \sqrt[4]{3}T_0 = \sqrt[4]{3}(293\ K) = \boxed{386\ K}$$

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27. SSM ***REASONING AND SOLUTION*** The net power generated by the stove is given by Equation 13.3, $P_{net} = e\sigma A(T^4 - T_0^4)$. Solving for T gives

$$T = \left(\frac{P_{net}}{e\sigma A} + T_0^4\right)^{1/4}$$

$$= \left\{\frac{7300\ W}{(0.900)[5.67\times10^{-8}\ J/(s\cdot m^2\cdot K^4)](2.00\ m^2)} + (302\ K)^4\right\}^{1/4} = \boxed{532\ K}$$

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28. ***REASONING*** According to the Stefan-Boltzmann law, the power radiated by an object is $Q/t = e\sigma T^4 A$ (Equation 13.2), where e is the emissivity of the object, σ is the Stefan-Boltzmann constant, T is the temperature (in kelvins) of the object, and A is the surface area of the object. The surface area of a sphere is $A = 4\pi R^2$, where R is the radius. Substituting this expression for A into Equation 13.2, and solving for the radius yields

$$R = \sqrt{\frac{\frac{Q}{t}}{4\pi e\sigma T^4}} \tag{1}$$

This expression will be used to find the radius of Sirius B.

SOLUTION Writing Equation (1) for both stars, we have

$$R_{Sirius} = \sqrt{\frac{\left(\frac{Q}{t}\right)_{Sirius}}{4\pi e_{Sirius}\sigma T_{Sirius}^4}} \quad \text{and} \quad R_{Sun} = \sqrt{\frac{\left(\frac{Q}{t}\right)_{Sun}}{4\pi e_{Sun}\sigma T_{Sun}^4}}$$

SOLUTION The radiant power produced by the sun is $Q/t = 3.9 \times 10^{26}$ W. The surface area of a sphere of radius r is $A = 4\pi r^2$. Since the sun is a perfect blackbody, $e = 1$. Solving Equation 13.2 for the surface temperature of the sun gives

$$T = \sqrt[4]{\frac{Q/t}{e\sigma 4\pi r^2}} = \sqrt[4]{\frac{3.9\times10^{26}\ \text{W}}{(1)\left[5.67\times10^{-8}\ \text{J/}\left(\text{s}\cdot\text{m}^2\cdot\text{K}^4\right)\right]4\pi\left(6.96\times10^{8}\ \text{m}\right)^2}} = \boxed{5800\ \text{K}}$$

38. ***REASONING*** The net rate at which energy is being lost via radiation cannot exceed the production rate of 115 J/s, if the body temperature is to remain constant. The net rate at which an object at temperature T radiates energy in a room where the temperature is T_0 is given by Equation 13.3 as $P_{net} = e\sigma A(T^4 - T_0^4)$. P_{net} is the net energy per second radiated. We need only set P_{net} equal to 115 J/s and solve for T_0. We note that the temperatures in this equation must be expressed in Kelvins, not degrees Celsius.

SOLUTION According to Equation 13.3, we have

$$P_{net} = e\sigma A\left(T^4 - T_0^4\right) \quad \text{or} \quad T_0^4 = T^4 - \frac{P_{net}}{e\sigma A}$$

Using Equation 12.1 to convert from degrees Celsius to Kelvins, we have $T = 34 + 273 = 307$ K. Using this value, it follows that

$$T_0 = \sqrt[4]{T^4 - \frac{P_{net}}{e\sigma A}}$$

$$= \sqrt[4]{(307\ \text{K})^4 - \frac{115\ \text{J/s}}{0.700\left[5.67\times10^{-8}\ \text{J/}\left(\text{s}\cdot\text{m}^2\cdot\text{K}^4\right)\right]\left(1.40\ \text{m}^2\right)}} = \boxed{287\ \text{K}\ (14\ °\text{C})}$$

39. ***REASONING AND SOLUTION*** The heat Q conducted during a time t through a wall of thickness L and cross sectional area A is given by Equation 13.1:

$$Q = \frac{kA\Delta T\, t}{L}$$

The radiant energy Q, emitted in a time t by a wall that has a Kelvin temperature T, surface area A, and emissivity e is given by Equation (13.2):

$$Q = e\sigma T^4 At$$

If the amount of radiant energy emitted per second per square meter at 0 °C is the same as the heat lost per second per square meter due to conduction, then

$$\left(\frac{Q}{tA}\right)_{\text{conduction}} = \left(\frac{Q}{tA}\right)_{\text{radiation}}$$

Making use of Equations 13.1 and 13.2, the equation above becomes

$$\frac{k\Delta T}{L} = e\sigma T^4$$

Solving for the emissivity e gives:

$$e = \frac{k\Delta T}{L\sigma T^4} = \frac{[1.1\ \text{J/(s}\cdot\text{m}\cdot\text{K)}](293.0\ \text{K} - 273.0\ \text{K})}{(0.10\ \text{m})[5.67\times10^{-8}\ \text{J/(s}\cdot\text{m}^2\cdot\text{K}^4)]\ (273.0\ \text{K})^4} = \boxed{0.70}$$

Remark on units: Notice that the units for the thermal conductivity were expressed as J/(s·m·K) even though they are given in Table 13.1 as J/(s·m·C°). The two units are equivalent since the "size" of a Celsius degree is the same as the "size" of a Kelvin; that is, 1 C° = 1 K. Kelvins were used, rather than Celsius degrees, to ensure consistency of units. However, Kelvins must be used in Equation 13.2 or any equation that is derived from it.

40. ***REASONING AND SOLUTION*** According to Equation 13.2, for the sphere we have $Q/t = e\sigma A_s T_s^4$, and for the cube $Q/t = e\sigma A_c T_c^4$. Equating and solving we get

$$T_c^4 = (A_s/A_c)T_s^4$$

Now

$$A_s/A_c = (4\pi R^2)/(6L^2)$$

The volume of the sphere and the cube are the same, (4/3) $\pi R^3 = L^3$, so $R = \left(\frac{3}{4\pi}\right)^{1/3} L$.

The ratio of the areas is $\frac{A_s}{A_c} = \frac{4\pi R^2}{6L^2} = \frac{4\pi}{6}\left(\frac{3}{4\pi}\right)^{2/3} = 0.806$. The temperature of the cube is, then

$$T_c = \left(\frac{A_s}{A_c}\right)^{1/4} T_s = (0.806)^{1/4}(773\ \text{K}) = \boxed{732\ \text{K}}$$