11. ***REASONING*** When a gas expands under isobaric conditions, its pressure remains constant. The work W done by the expanding gas is $W = P(V_f - V_i)$, Equation 15.2, where P is the pressure and V_f and V_i are the final and initial volumes. Since all the variables in this relation are known, we can solve for the final volume.

SOLUTION Solving $W = P(V_f - V_i)$ for the final volume gives

$$V_f = \frac{W}{P} + V_i = \frac{480\ \text{J}}{1.6\times10^5\ \text{Pa}} + 1.5\times10^{-3}\ \text{m}^3 = \boxed{4.5\times10^{-3}\ \text{m}^3}$$

12. ***REASONING*** The work done in the process is equal to the "area" under the curved line between A and B in the drawing. From the graph, we find that there are about 78 "squares" under the curve. Each square has an "area" of

$$\left(2.0\times10^4\ \text{Pa}\right)\left(2.0\times10^{-3}\ \text{m}^3\right) = 4.0\times10^1\ \text{J}$$

SOLUTION

a. The work done in the process has a magnitude of

$$W = (78)\left(4.0\times10^1\ \text{J}\right) = \boxed{3100\ \text{J}}$$

b. The final volume is smaller than the initial volume, so the gas is compressed. Therefore, work is done on the gas so the work is negative.

13. ***REASONING*** For segment AB, there is no work, since the volume is constant. For segment BC the process is isobaric and Equation 15.2 applies. For segment CA, the work can be obtained as the area under the line CA in the graph.

SOLUTION

a. For segment AB, the process is isochoric, that is, the volume is constant. For a process in which the volume is constant, no work is done, so $\boxed{W = 0\ \text{J}}$.

b. For segment BC, the process is isobaric, that is, the pressure is constant. Here, the volume is increasing, so the gas is expanding against the outside environment. As a result, the gas does work, which is positive according to our convention. Using Equation 15.2 and the data in the drawing, we obtain

$$W = P\left(V_f - V_i\right)$$

$$= \left(7.0\times10^5\ \text{Pa}\right)\left[\left(5.0\times10^{-3}\ \text{m}^3\right) - \left(2.0\times10^{-3}\ \text{m}^3\right)\right] = \boxed{+2.1\times10^3\ \text{J}}$$

c. For segment CA, the volume of the gas is decreasing, so the gas is being compressed and work is being done on it. Therefore, the work is negative, according to our convention. The magnitude of the work is the area under the segment CA. We estimate that this area is 15 of the squares in the graphical grid. The area of each square is

$$(1.0\times10^5\ \text{Pa})(1.0\times10^{-3}\ \text{m}^3) = 1.0\times10^2\ \text{J}$$

The work, then, is

$$W = -15\ (1.0\times10^2\ \text{J}) = \boxed{-1.5\times10^3\ \text{J}}$$

14. ***REASONING*** The pressure P of the gas remains constant while its volume increases by an amount ΔV. Therefore, the work W done by the expanding gas is given by $W = P\Delta V$ (Equation 15.2). ΔV is known, so if we can obtain a value for W, we can use this expression to calculate the pressure. To determine W, we turn to the first law of thermodynamics $\Delta U = Q - W$ (Equation 15.1), where Q is the heat and ΔU is the change in the internal energy. ΔU is given, so to use the first law to determine W we need information about Q. According to Equation 12.4, the heat needed to raise the temperature of a mass m of material by an amount ΔT is $Q = cm\Delta T$ where c is the material's specific heat capacity.

SOLUTION According to Equation 15.2, the pressure P of the expanding gas can be determined from the work W and the change ΔV in volume of the gas according to

$$P = \frac{W}{\Delta V}$$

Using the first law of thermodynamics, we can write the work as $W = Q - \Delta U$ (Equation 15.1). With this substitution, the expression for the pressure becomes

$$P = \frac{W}{\Delta V} = \frac{Q - \Delta U}{\Delta V} \qquad (1)$$

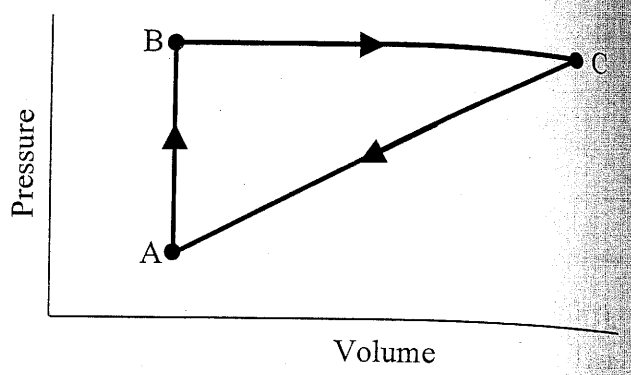
Using Equation 12.4, we can write the heat as $Q = cm\Delta T$, which can then be substituted into Equation (1). Thus,

$$P = \frac{Q - \Delta U}{\Delta V} = \frac{cm\Delta T - \Delta U}{\Delta V}$$

$$= \frac{\left[1080\ \text{J/(kg}\cdot\text{C°)}\right]\left(24.0\times10^{-3}\ \text{kg}\right)\left(53.0\ \text{C°}\right) - 939\ \text{J}}{1.40\times10^{-3}\ \text{m}^3} = \boxed{3.1\times10^5\ \text{Pa}}$$

28. ***REASONING*** The first law of thermodynamics states that due to heat Q and work W, the internal energy of the system changes by an amount ΔU according to $\Delta U = Q - W$ (Equation 15.1). This law will enable us to find the various quantities for the three processes.

Process A→B There is no work done for the process A→B. The reason is that the volume is constant (see the drawing), which means that the change ΔV in the volume is zero. Thus, the area under the plot of pressure versus volume is zero, and $W_{A\to B} = 0$ J. Since Q is known, the first law can then be used to find ΔU.



Process B→C Since the change $\Delta U_{B\to C}$ in the internal energy of the gas and the work $W_{B\to C}$ are known for the process B→C, the heat $Q_{B\to C}$ can be determined directly by using the first law of thermodynamics: $Q_{B\to C} = \Delta U_{B\to C} + W_{B\to C}$ (Equation 15.1).

Process C→A For the process C→A it is possible to find the change in the internal energy of the gas once the changes in the internal energy for the processes A→B and B→C are known. The total change ΔU_{total} in the internal energy for the three processes is $\Delta U_{total} = \Delta U_{A\to B} + \Delta U_{B\to C} + U_{C\to A}$, and we can use this equation to find $\Delta U_{C\to A}$, with the aid of values for $\Delta U_{A\to B}$ and $\Delta U_{B\to C}$. This is possible because ΔU_{total} is the change in the internal energy for the total process A→B→C→A. This process begins and ends at the same place on the pressure-versus-volume plot. Therefore, the value of the internal energy U is the same at the start and the end, with the result that $\Delta U_{total} = 0$ J.

SOLUTION

a. Since the change in volume is zero ($\Delta V = 0$ m^3), the area under the plot of pressure versus volume is zero, with the result that the work is $W_{A\to B} =$ 0 J.

b. The change in the internal energy of the gas for the process A→B is found from the first law of thermodynamics:

$$\Delta U_{A\to B} = Q_{A\to B} - W_{A\to B} = +561 \text{ J} - 0 \text{ J} = \boxed{+561 \text{ J}}$$

c. According to the first law of thermodynamics, the change in the internal energy of the gas for the process B→C is $\Delta U_{B\to C} = Q_{B\to C} - W_{B\to C}$. Thus, the heat for this process is

$$Q_{B\to C} = \Delta U_{B\to C} + W_{B\to C} = +4303 \text{ J} + 3740 \text{ J} = \boxed{+8043 \text{ J}}$$

Since this heat is positive, it is added to the gas.

d. The change ΔU_{total} in the total internal energy for the three processes is $\Delta U_{total} = \Delta U_{A\to B} + \Delta U_{B\to C} + \Delta U_{C\to A}$. Solving this equation for $\Delta U_{C\to A}$ gives $\Delta U_{C\to A} = \Delta U_{total} - \Delta U_{A\to B} - \Delta U_{B\to C}$. As discussed in the ***REASONING*** **(Process C→A)**, $\Delta U_{total} = 0$ J, since the third process ends at point A, which is the start of the first process. Therefore,

$$\Delta U_{C\to A} = 0 \text{ J} - 561 \text{ J} - 4303 \text{ J} = \boxed{-4864 \text{ J}}$$

e. According to the first law of thermodynamics, the change in the internal energy of the gas for the process C→A is $\Delta U_{C\to A} = Q_{C\to A} - W_{C\to A}$. The heat for this process is

$$Q_{C\to A} = \Delta U_{C\to A} + W_{C\to A} = -4864 \text{ J} + (-2867 \text{ J}) = \boxed{-7731 \text{ J}}$$

Since this heat is negative, it is removed from the gas.

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29. SSM ***REASONING AND SOLUTION***

Step A → B

The internal energy of a monatomic ideal gas is $U = (3/2)nRT$. Thus, the change is

$$\Delta U = \tfrac{3}{2} nR\Delta T = \tfrac{3}{2}(1.00 \text{ mol})\left[8.31 \text{ J/(mol·K)}\right](800.0 \text{ K} - 400.0 \text{ K}) = \boxed{4990 \text{ J}}$$

The work for this constant pressure step is $W = P\Delta V$. But the ideal gas law applies, so

$$W = P\Delta V = nR\Delta T = (1.00 \text{ mol})\left[8.31 \text{ J/(mol·K)}\right](800.0 \text{ K} - 400.0 \text{ K}) = \boxed{3320 \text{ J}}$$

The first law of thermodynamics indicates that the heat is

$$Q = \Delta U + W = \tfrac{3}{2} nR\Delta T + nR\Delta T$$

$$= \tfrac{5}{2}(1.00 \text{ mol})\left[8.31 \text{ J/(mol·K)}\right](800.0 \text{ K} - 400.0 \text{ K}) = \boxed{8310 \text{ J}}$$

Step B → C

The internal energy of a monatomic ideal gas is $U = (3/2)nRT$. Thus, the change is

$$\Delta U = \tfrac{3}{2} nR\Delta T = \tfrac{3}{2}(1.00 \text{ mol})\left[8.31 \text{ J/(mol·K)}\right](400.0 \text{ K} - 800.0 \text{ K}) = \boxed{-4990 \text{ J}}$$

The volume is constant in this step, so the work done by the gas is $\boxed{W = 0 \text{ J}}$.

The first law of thermodynamics indicates that the heat is

$$Q = \Delta U + W = \Delta U = \boxed{-4990 \text{ J}}$$

Step C → D

The internal energy of a monatomic ideal gas is $U = (3/2)nRT$. Thus, the change is

$$\Delta U = \tfrac{3}{2}nR\Delta T = \tfrac{3}{2}(1.00 \text{ mol})\left[8.31 \text{ J/(mol}\cdot\text{K)}\right](200.0 \text{ K} - 400.0 \text{ K}) = \boxed{-2490 \text{ J}}$$

The work for this constant pressure step is $W = P\Delta V$. But the ideal gas law applies, so

$$W = P\Delta V = nR\Delta T = (1.00 \text{ mol})\left[8.31 \text{ J/(mol}\cdot\text{K)}\right](200.0 \text{ K} - 400.0 \text{ K}) = \boxed{-1660 \text{ J}}$$

The first law of thermodynamics indicates that the heat is

$$Q = \Delta U + W = \tfrac{3}{2}nR\Delta T + nR\Delta T$$

$$= \tfrac{5}{2}(1.00 \text{ mol})\left[8.31 \text{ J/(mol}\cdot\text{K)}\right](200.0 \text{ K} - 400.0 \text{ K}) = \boxed{-4150 \text{ J}}$$

Step D → A

The internal energy of a monatomic ideal gas is $U = (3/2)nRT$. Thus, the change is

$$\Delta U = \tfrac{3}{2}nR\Delta T = \tfrac{3}{2}(1.00 \text{ mol})\left[8.31 \text{ J/(mol}\cdot\text{K)}\right](400.0 \text{ K} - 200.0 \text{ K}) = \boxed{2490 \text{ J}}$$

The volume is constant in this step, so the work done by the gas is $\boxed{W = 0 \text{ J}}$

The first law of thermodynamics indicates that the heat is

$$Q = \Delta U + W = \Delta U = \boxed{2490 \text{ J}}$$

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30. ***REASONING*** The cylinder containing the gas is perfectly insulated, so no heat can enter or leave. Therefore, the compression of the gas is adiabatic, and the initial and final pressures (P_i, P_f) and volumes (V_i, V_f) of the gas are related by $P_iV_i^{\gamma} = P_fV_f^{\gamma}$ (Equation 15.5), where $\gamma = \frac{5}{3}$ because the gas is monatomic and ideal. The final Kelvin temperature T_f of the gas is twice the initial temperature T_i, so we have that $T_f = 2T_i$. We will use the ideal gas law $PV = nRT$ (Equation 14.1) to determine the final pressure P_f of the gas from its volume and Kelvin temperature. In Equation 14.1, n is the number of moles of the gas and $R = 8.31$ J/(mol·K) is the universal gas constant.

SOLUTION In order to make use of the ideal gas law $PV = nRT$ (Equation 14.1), it is convenient to rewrite $P_iV_i^{\gamma} = P_fV_f^{\gamma}$ (Equation 15.5) so that the product PV is raised to the power γ:

$$P_i^{1-\gamma}P_i^{\gamma}V_i^{\gamma} = P_f^{1-\gamma}P_f^{\gamma}V_f^{\gamma} \quad \text{or} \quad P_i^{1-\gamma}(P_iV_i)^{\gamma} = P_f^{1-\gamma}(P_fV_f)^{\gamma} \tag{1}$$

In Equation (1), we have used the fact that $P^{(a+b)} = P^aP^b$. Substituting $P_iV_i = nRT_i$ and $P_fV_f = nRT_f$ (Equation 14.1) into Equation (1) yields

$$P_i^{1-\gamma}(nRT_i)^{\gamma} = P_f^{1-\gamma}(nRT_f)^{\gamma} \quad \text{or} \quad P_i^{1-\gamma}T_i^{\gamma} = P_f^{1-\gamma}T_f^{\gamma} \tag{2}$$

Solving Equation (2) for $P_f^{1-\gamma}$ and then raising both sides to the power $1/(1-\gamma)$, we obtain

$$P_f^{1-\gamma} = \frac{P_i^{1-\gamma}T_i^{\gamma}}{T_f^{\gamma}} = P_i^{1-\gamma}\left(\frac{T_i}{T_f}\right)^{\gamma} \quad \text{or} \quad P_f = P_i\left(\frac{T_i}{T_f}\right)^{\gamma/(1-\gamma)} \tag{3}$$

Equation (3) makes use of the fact that $(P^a)^{1/a} = P^{a/a} = P^1 = P$. Substituting $T_f = 2T_i$ and $\gamma = \frac{5}{3}$ into Equation (3), we find that

$$P_f = (1.50\times10^5 \text{ Pa})\left(\frac{T_i}{2T_i}\right)^{(5/3)/[1-(5/3)]} = (1.50\times10^5 \text{ Pa})\left(\frac{1}{2}\right)^{-(5/2)} = \boxed{8.49\times10^5 \text{ Pa}}$$

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31. ***REASONING AND SOLUTION***

a. Since the curved line between A and C is an isotherm, the initial and final temperatures are the same. Since the internal energy of an ideal monatomic gas is $U = (3/2)nRT$, the initial and final energies are also the same, and the change in the internal energy is $\Delta U = 0$. The first law of thermodynamics, then, indicates that for the process A→B→C, we have

$$\Delta U = 0 = Q - W \quad \text{or} \quad Q = W$$

The heat is equal to the work. Determining the work from the area beneath the straight line segments AB and BC, we find that

$$Q = W = -(4.00\times10^5 \text{ Pa})(0.400 \text{ m}^3 - 0.200 \text{ m}^3) = \boxed{-8.00\times10^4 \text{ J}}$$

b. The minus sign is included because the gas is compressed, so that work is done on the gas. Since the answer for Q is negative, we conclude that heat flows out of the gas.

32. ***REASONING*** We seek the difference W_{diff} in work between the work W_{ad} done by the gas during the adiabatic process and the work W_{alt} done by the gas during the alternative process:

$$W_{\text{diff}} = W_{\text{ad}} - W_{\text{alt}} \tag{1}$$

Considering the alternative process first, we bear in mind that, according to $W = P(V_{\text{f}} - V_{\text{i}})$ (Equation 15.2), a gas only does work if its volume increases. As the pressure of the gas decreases to P_{f} from P_{i} during the first step, its volume remains constant, and, therefore, the gas does no work. Consequently, the total work W_{alt} done by the gas during the entire alternative process occurs during the second step, when the gas expands to a volume V_{f} from a volume V_{i} at a constant pressure P_{f}:

$$W_{\text{alt}} = P_{\text{f}}(V_{\text{f}} - V_{\text{i}}) \tag{2}$$

Only the initial volume V_{i} is given, so the final volume V_{f} in Equation (2) must be determined. Because the gas is ideal and monatomic, and can undergo an adiabatic process that takes it from the initial volume V_{i} to the final volume V_{f}, we will use $P_{\text{i}}V_{\text{i}}^{\gamma} = P_{\text{f}}V_{\text{f}}^{\gamma}$ (Equation 15.5), with $\gamma = \frac{5}{3}$, to determine the final volume V_{f}. Solving Equation 15.5 for the final volume yields $V_{\text{f}}^{\gamma} = \dfrac{P_{\text{i}}V_{\text{i}}^{\gamma}}{P_{\text{f}}}$, or

$$V_{\text{f}} = \left(\frac{P_{\text{i}}V_{\text{i}}^{\gamma}}{P_{\text{f}}}\right)^{1/\gamma} = V_{\text{i}}\left(\frac{P_{\text{i}}}{P_{\text{f}}}\right)^{1/\gamma} = \left(6.34\times10^{-3}\ \text{m}^3\right)\left(\frac{2.20\times10^{5}\ \text{Pa}}{8.15\times10^{4}\ \text{Pa}}\right)^{\frac{3}{5}} = 1.15\times10^{-2}\ \text{m}^3$$

In the adiabatic process, the gas does an amount of work W_{ad} given by

$$W_{\text{ad}} = \tfrac{3}{2}nR(T_{\text{i}} - T_{\text{f}}) \tag{15.4}$$

In Equation 15.4, n is the number of moles of the gas, R is the universal gas constant, and T_{i} and T_{f} are the initial and final Kelvin temperatures of the gas. In order to determine the initial and final temperatures, we will employ the ideal gas law: $PV = nRT$ (Equation 14.1). Solving Equation 14.1 for the initial and final Kelvin temperatures yields

$$T_{\text{i}} = \frac{P_{\text{i}}V_{\text{i}}}{nR} \qquad \text{and} \qquad T_{\text{f}} = \frac{P_{\text{f}}V_{\text{f}}}{nR} \tag{3}$$

SOLUTION Substituting Equations (3) into Equation 15.4 gives

$$W_{\text{ad}} = \tfrac{3}{2}nR(T_{\text{i}} - T_{\text{f}}) = \tfrac{3}{2}\cancel{nR}\left(\frac{P_{\text{i}}V_{\text{i}}}{\cancel{nR}} - \frac{P_{\text{f}}V_{\text{f}}}{\cancel{nR}}\right) = \tfrac{3}{2}(P_{\text{i}}V_{\text{i}} - P_{\text{f}}V_{\text{f}}) \tag{4}$$

Substituting Equations (4) and (2) into Equation (1), we obtain

$$\begin{aligned} W_{\text{diff}} &= W_{\text{ad}} - W_{\text{alt}} = \tfrac{3}{2}(P_{\text{i}}V_{\text{i}} - P_{\text{f}}V_{\text{f}}) - P_{\text{f}}(V_{\text{f}} - V_{\text{i}}) \\ &= \tfrac{3}{2}P_{\text{i}}V_{\text{i}} - \tfrac{3}{2}P_{\text{f}}V_{\text{f}} - P_{\text{f}}V_{\text{f}} + P_{\text{f}}V_{\text{i}} = \tfrac{3}{2}P_{\text{i}}V_{\text{i}} - \tfrac{5}{2}P_{\text{f}}V_{\text{f}} + P_{\text{f}}V_{\text{i}} \\ &= \tfrac{3}{2}P_{\text{i}}V_{\text{i}} + P_{\text{f}}\left(V_{\text{i}} - \tfrac{5}{2}V_{\text{f}}\right) \end{aligned}$$

Substituting in the given values of P_{i}, P_{f}, V_{i}, and the calculated value of V_{f}, yields

$$W_{\text{diff}} = \tfrac{3}{2}\left(2.20\times10^{5}\ \text{Pa}\right)\left(6.34\times10^{-3}\ \text{m}^3\right) + \left(8.15\times10^{4}\ \text{Pa}\right)\left[6.34\times10^{-3}\ \text{m}^3 - \tfrac{5}{2}\left(1.15\times10^{-2}\ \text{m}^3\right)\right]$$

$$= 270\ \text{J}$$

33. SSM ***REASONING AND SOLUTION*** Let the left be side 1 and the right be side 2. Since the partition moves to the right, side 1 does work on side 2, so that the work values involved satisfy the relation $W_1 = -W_2$. Using Equation 15.4 for each work value, we find that

$$\tfrac{3}{2}nR(T_{1\text{i}} - T_{1\text{f}}) = -\tfrac{3}{2}nR(T_{2\text{i}} - T_{2\text{f}}) \qquad \text{or}$$

$$T_{1\text{f}} + T_{2\text{f}} = T_{1\text{i}} + T_{2\text{i}} = 525\ \text{K} + 275\ \text{K} = 8.00\times10^{2}\ \text{K}$$

We now seek a second equation for the two unknowns $T_{1\text{f}}$ and $T_{2\text{f}}$. Equation 15.5 for an adiabatic process indicates that $P_{1\text{i}}V_{1\text{i}}^{\gamma} = P_{1\text{f}}V_{1\text{f}}^{\gamma}$ and $P_{2\text{i}}V_{2\text{i}}^{\gamma} = P_{2\text{f}}V_{2\text{f}}^{\gamma}$. Dividing these two equations and using the facts that $V_{1\text{i}} = V_{2\text{i}}$ and $P_{1\text{f}} = P_{2\text{f}}$, gives

$$\frac{P_{1\text{i}}V_{1\text{i}}^{\gamma}}{P_{2\text{i}}V_{2\text{i}}^{\gamma}} = \frac{P_{1\text{f}}V_{1\text{f}}^{\gamma}}{P_{2\text{f}}V_{2\text{f}}^{\gamma}} \qquad \text{or} \qquad \frac{P_{1\text{i}}}{P_{2\text{i}}} = \left(\frac{V_{1\text{f}}}{V_{2\text{f}}}\right)^{\gamma}$$

Using the ideal gas law, we find that