

**SOLUTION** According to Equation 3.2, we have

$$a = \frac{v - v_0}{t - t_0} \quad \text{or} \quad 340 \text{ m/s}^2 = \frac{v - 0 \text{ m/s}}{0.050 \text{ s}} \quad \text{or} \quad v = (340 \text{ m/s}^2)(0.050 \text{ s})$$

Using trigonometry, we find the components to be

$$v_x = v \cos 51^\circ = (340 \text{ m/s}^2)(0.050 \text{ s}) \cos 51^\circ = \boxed{11 \text{ m/s}}$$

$$v_y = v \sin 51^\circ = (340 \text{ m/s}^2)(0.050 \text{ s}) \sin 51^\circ = \boxed{13 \text{ m/s}}$$

4. **REASONING** The meteoroid's speed is the magnitude of its velocity vector, here described in terms of two perpendicular components, one directed toward the east and one directed vertically downward. Let east be the  $+x$  direction, and up be the  $+y$  direction. Then the components of the meteoroid's velocity are  $v_x = +18.3 \text{ km/s}$  and  $v_y = -11.5 \text{ km/s}$ . The meteoroid's speed  $v$  is related to these components by the Pythagorean theorem (Equation 1.7):  $v^2 = v_x^2 + v_y^2$ .

**SOLUTION** From the Pythagorean theorem,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(+18.3 \text{ km/s})^2 + (-11.5 \text{ km/s})^2} = \boxed{21.6 \text{ km/s}}$$

It's important to note that the negative sign for  $v_y$  becomes a positive sign when this quantity is squared. Forgetting this fact would yield a value for  $v$  that is smaller than  $v_x$ , but the magnitude of a vector cannot be smaller than either of its components.

5. **SSM REASONING AND SOLUTION**

$$x = r \cos \theta = (162 \text{ km}) \cos 62.3^\circ = \boxed{75.3 \text{ km}}$$

$$y = r \sin \theta = (162 \text{ km}) \sin 62.3^\circ = \boxed{143 \text{ km}}$$

6. **REASONING AND SOLUTION** The horizontal displacement is

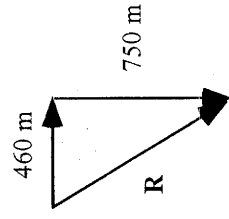
$$x = 19\,600 \text{ m} - 11\,200 \text{ m} = 8400 \text{ m}$$

The vertical displacement is

$$y = 4900 \text{ m} - 3200 \text{ m} = 1700 \text{ m}$$

The magnitude of the displacement is therefore,

$$\Delta r = \sqrt{x^2 + y^2} = \sqrt{(8400 \text{ m})^2 + (1700 \text{ m})^2} = \boxed{8600 \text{ m}}$$



7. **SSM REASONING** The displacement of the elephant seal has two components; 460 m due east and 750 m downward. These components are mutually perpendicular; hence, the Pythagorean theorem can be used to determine their resultant.

**SOLUTION** From the Pythagorean theorem,

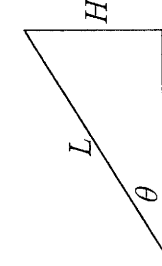
$$R^2 = (460 \text{ m})^2 + (750 \text{ m})^2$$

Therefore,

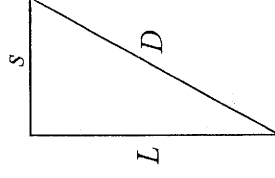
$$R = \sqrt{(460 \text{ m})^2 + (750 \text{ m})^2} = \boxed{8.8 \times 10^2 \text{ m}}$$

8. **REASONING** Consider first the shopper's ride up the escalator. Let the diagonal length of the escalator be  $L$ , the height of the upper floor be  $H$ , and the angle that the escalator makes with respect to the horizontal be  $\theta$  (see the diagram). Because  $L$  is the hypotenuse of the right triangle and  $H$  is opposite the angle  $\theta$ , the three quantities are related by the inverse sine function:

$$\theta = \sin^{-1} \left( \frac{h_o}{h} \right) = \sin^{-1} \left( \frac{H}{L} \right) \quad (1.4)$$



Up the escalator



Entire view

Now consider the entire trip from the bottom to the top of the escalator (a distance  $L$ ), and then from the top of the escalator to the store entrance (a distance  $s$ ). The right turn between these two parts of the trip means that they are perpendicular (see the diagram). The shopper's total displacement has a magnitude  $D$ , and this serves as the hypotenuse of a right triangle with  $L$  and  $s$ . From the Pythagorean theorem, the three sides are related as follows:  $D^2 = L^2 + s^2$ .

**SOLUTION** Solving  $D^2 = L^2 + s^2$  for the length  $L$  of the escalator gives  $L = \sqrt{D^2 - s^2}$ . We now use this result and the relation  $\theta = \sin^{-1} \left( \frac{H}{L} \right)$  to obtain the angle  $\theta$ :

39. **REASONING** The speed  $v$  of the soccer ball just before the goalie catches it is given by  $v = \sqrt{v_x^2 + v_y^2}$ , where  $v_x$  and  $v_y$  are the  $x$  and  $y$  components of the final velocity of the ball. The data for this problem are (the  $+x$  direction is from the kicker to the goalie, and the  $+y$  direction is the "up" direction):

***x*-Direction Data**

$x$	$a_x$	$v_x$	$v_{0x}$	$t$
+16.8 m	0 m/s <sup>2</sup>	?	$+(16.0 \text{ m/s}) \cos 28.0^\circ = +14.1 \text{ m/s}$	

***y*-Direction Data**

$y$	$a_y$	$v_y$	$v_{0y}$	$t$
	-9.80 m/s <sup>2</sup>	?	$+(16.0 \text{ m/s}) \sin 28.0^\circ = +7.51 \text{ m/s}$	

Since there is no acceleration in the  $x$  direction ( $a_x = 0 \text{ m/s}^2$ ),  $v_x$  remains the same as  $v_{0x}$ , so  $v_x = v_{0x} = +14.1 \text{ m/s}$ . The time  $t$  that the soccer ball is in the air can be found from the  $x$ -direction data, since three of the variables are known. With this value for the time and the  $y$ -direction data, the  $y$  component of the final velocity can be determined.

**SOLUTION** Since  $a_x = 0 \text{ m/s}^2$ , the time can be calculated from Equation 3.5a as  $t = \frac{x}{v_{0x}} = \frac{+16.8 \text{ m}}{+14.1 \text{ m/s}} = 1.19 \text{ s}$ . The value for  $v_y$  can now be found by using Equation 3.3b with this value of the time and the  $y$ -direction data:

$$v_y = v_{0y} + a_y t = +7.51 \text{ m/s} + (-9.80 \text{ m/s}^2)(1.19 \text{ s}) = -4.15 \text{ m/s}$$

The speed of the ball just as it reaches the goalie is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(+14.1 \text{ m/s})^2 + (-4.15 \text{ m/s})^2} = \boxed{14.7 \text{ m/s}}$$

40. **REASONING AND SOLUTION** On impact

$$v_x = v \cos 75.0^\circ = (8.90 \text{ m/s}) \cos 75.0^\circ = 2.30 \text{ m/s}$$

and

$$v_{0y}^2 = v_y^2 + 2gy = (8.90 \text{ m/s})^2 \sin^2 75.0^\circ + 2(9.80 \text{ m/s}^2)(-3.00 \text{ m})$$

so

$$v_{0y} = 3.89 \text{ m/s}$$

The magnitude of the diver's initial velocity is

$$v_0 = \sqrt{(2.30 \text{ m/s})^2 + (3.89 \text{ m/s})^2} = \boxed{4.52 \text{ m/s}}$$

The angle the initial velocity vector makes with the horizontal is

$$\theta_0 = \tan^{-1}(v_{0y}/v_{0x}) = \boxed{59.4^\circ}$$

41. **REASONING** As discussed in Conceptual Example 5, the horizontal velocity component of the bullet does not change from its initial value and is equal to the horizontal velocity of the car. The same thing is true here for the tomato. In other words, regardless of its vertical position relative to the ground, the tomato always remains above you as you travel in the convertible. From the symmetry of free fall motion, we know that when you catch the tomato, its velocity will be 11 m/s straight downward. The time  $t$  required to catch the tomato can be found by solving Equation 3.3b ( $v_y = v_{0y} + a_y t$ ) with  $v_y = -v_{0y}$ . Once  $t$  is known, the distance that the car moved can be found from  $x = v_x t$ .

**SOLUTION** Taking upward as the positive direction, we find the flight time of the tomato to be

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-2v_{0y}}{a_y} = \frac{-2(11 \text{ m/s})}{-9.80 \text{ m/s}^2} = 2.24 \text{ s}$$

Thus, the car moves through a distance of

$$x = v_x t = (25 \text{ m/s})(2.24 \text{ s}) = \boxed{56 \text{ m}}$$

42. **REASONING** As shown in the drawing, the angle that the velocity vector makes with the horizontal is given by

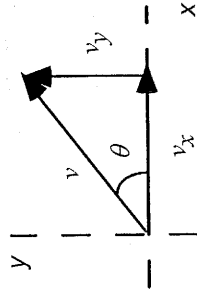
$$\tan \theta = \frac{v_y}{v_x}$$

where, from Equation 3.3b,

$$v_y = v_{0y} + a_y t = v_0 \sin \theta_0 + a_y t$$

and, from Equation 3.3a (since  $a_x = 0 \text{ m/s}^2$ ),

$$v_x = v_{0x} = v_0 \cos \theta_0$$



$$\sin 2\theta_2 = 2.00 \sin 2\theta_1 = 2.00 \sin 2(12.0^\circ) = 0.813$$

$$\theta_2 = \frac{\sin^{-1} 0.813}{2.00} = \boxed{27.2^\circ}$$

47. **SSM REASONING AND SOLUTION** In the absence of air resistance, the bullet exhibits projectile motion. The  $x$  component of the motion has zero acceleration while the  $y$  component of the motion is subject to the acceleration due to gravity. The horizontal distance traveled by the bullet is given by Equation 3.5a (with  $a_x = 0 \text{ m/s}^2$ ):

$$x = v_{0x}t = (v_0 \cos \theta)t$$

with  $t$  equal to the time required for the bullet to reach the target. The time  $t$  can be found by considering the vertical motion. From Equation 3.3b,

$$v_y = v_{0y} + a_y t$$

When the bullet reaches the target,  $v_y = -v_{0y}$ . Assuming that up and to the right are the positive directions, we have

$$t = \frac{-2v_{0y}}{a_y} = \frac{-2v_0 \sin \theta}{a_y} \quad \text{and} \quad x = (v_0 \cos \theta) \left( \frac{-2v_0 \sin \theta}{a_y} \right)$$

Using the fact that  $2 \sin \theta \cos \theta = \sin 2\theta$ , we have

$$x = -\frac{2v_0^2 \cos \theta \sin \theta}{a_y} = -\frac{v_0^2 \sin 2\theta}{a_y}$$

Thus, we find that

$$\sin 2\theta = -\frac{x a_y}{v_0^2} = \frac{(91.4 \text{ m})(-9.80 \text{ m/s}^2)}{(427 \text{ m/s})^2} = 4.91 \times 10^{-3}$$

and

$$2\theta = 0.281^\circ \quad \text{or} \quad 2\theta = 180.000^\circ - 0.281^\circ = 179.719^\circ$$

Therefore,

$$\theta = \boxed{0.141^\circ} \quad \text{and} \quad \boxed{89.860^\circ}$$

48. **REASONING** The angle  $\theta$  is the angle that the balloon's initial velocity  $v_0$  makes with the horizontal, and can be found from the horizontal and vertical components of the initial velocity:

$$\tan \theta = \frac{v_{0y}}{v_{0x}} \quad \text{or} \quad \theta = \tan^{-1} \left( \frac{v_{0y}}{v_{0x}} \right) \quad (1)$$

(Note: because the balloon follows a curved trajectory,  $\theta$  is not related in this fashion to the horizontal and vertical components  $x$  and  $y$  of the balloon's displacement.) We will use  $x = v_{0x}t + \frac{1}{2}a_x t^2$  (Equation 3.5a) and  $y = v_{0y}t + \frac{1}{2}a_y t^2$  (Equation 3.5b) to find expressions for the horizontal and vertical components ( $v_{0x}$  and  $v_{0y}$ ) of the balloon's initial velocity, and then combine those results with Equation (1) to find  $\theta$ . For the initial horizontal velocity component, with  $a_x = 0 \text{ m/s}^2$  since air resistance is being ignored, Equation 3.5a gives

$$x = v_{0x}t + \frac{1}{2}(0 \text{ m/s}^2)t^2 = v_{0x}t \quad \text{or} \quad v_{0x} = \frac{x}{t} \quad (2)$$

For the initial vertical velocity component, we obtain from Equation 3.5b that

$$y = v_{0y}t + \frac{1}{2}a_y t^2 \quad \text{or} \quad v_{0y} = \frac{y - \frac{1}{2}a_y t^2}{t} \quad (3)$$

In part *b*, the initial speed  $v_0$  of the second balloon can be found by first noting that it is related to the  $x$  component  $v_{0x}$  of the balloon's initial velocity by  $v_{0x} = v_0 \cos \theta$ . Since there is no acceleration in the  $x$  direction ( $a_x = 0 \text{ m/s}^2$ ),  $v_{0x}$  is equal to the  $x$  component of the balloon's displacement ( $x = +35.0 \text{ m}$ ) divided by the time  $t$  that the second balloon is in the air, or  $v_{0x} = x/t$ . Thus, the relation  $v_{0x} = v_0 \cos \theta$  can be written as

$$\frac{x}{t} = v_0 \cos \theta \quad (4)$$

In Equation (4),  $x = +35.0 \text{ m}$  and  $\theta$  is known from the result of part (a). The time of flight  $t$  is related to the  $y$  component of the balloon's displacement by  $y = v_{0y}t + \frac{1}{2}a_y t^2$  (Equation 3.5b). Since  $v_{0y} = v_0 \sin \theta$ , Equation 3.5b can be expressed as

$$y = v_0 \sin \theta t + \frac{1}{2}a_y t^2 = (v_0 \sin \theta)t + \frac{1}{2}a_y t^2 \quad (5)$$

Equations (4) and (5) provide everything necessary to find the initial speed  $v_0$  of the water balloon for the second launch.

$x = v_{0x}t + \frac{1}{2}a_x t^2$  (Equation 3.5a) to find the elapsed time. With  $a_x = 0 \text{ m/s}^2$ , Equation 3.5a becomes  $x = v_{0x}t + \frac{1}{2}(0 \text{ m/s}^2)t^2 = v_{0x}t$ . Thus the car is in the air for  $t$  seconds, where

$$t = \frac{x}{v_{0x}} = \frac{x}{v_0 \cos \theta} \quad (1)$$

In Equation (1),  $x = 15 \text{ m}$  and  $\theta = 16^\circ$ , but  $v_0$  and  $t$  are not known. The heights of the trailer and the ramp are the same, so the car's vertical displacement is zero. To use this fact, we turn to  $y = v_{0y}t + \frac{1}{2}a_y t^2$  (Equation 3.5b) and substitute both  $y = 0 \text{ m}$  and  $v_{0y} = v_0 \sin \theta$ :

$$0 = v_{0y}t + \frac{1}{2}a_y t^2 \quad \text{or} \quad v_{0y} = -\frac{1}{2}a_y t \quad \text{or} \quad v_0 \sin \theta = -\frac{1}{2}a_y t \quad (2)$$

To eliminate the time, we substitute  $t$  from Equation (1) into Equation (2) and find that

$$v_0 \sin \theta = -\frac{1}{2}a_y \left( \frac{x}{v_0 \cos \theta} \right) \quad \text{or} \quad v_0^2 = \frac{-a_y x}{2 \cos \theta \sin \theta} \quad \text{or} \quad v_0 = \sqrt{\frac{-a_y x}{2 \cos \theta \sin \theta}} \quad (3)$$

**SOLUTION** Taking up as the positive direction, we use Equation (3) to calculate the speed  $v_{\text{CT}} = v_0$  of the car relative to the trucks:

$$v_{\text{CT}} = v_0 = \sqrt{\frac{-a_y x}{2 \cos \theta \sin \theta}} = \sqrt{\frac{-(-9.80 \text{ m/s}^2)(15 \text{ m})}{2 \cos 16^\circ \sin 16^\circ}} = 17 \text{ m/s}$$

Since the ramp alters the direction of the car's velocity but not its magnitude, the initial jump speed  $v_0$  is also the magnitude of the car's velocity  $v_{\text{CT}}$  relative to the truck before the car reaches the ramp:  $v_0 = v_{\text{CT}}$ . The sports car must overtake the truck at a speed of at least  $17 \text{ m/s}$  relative to the truck, so that the car's minimum required speed  $v_{\text{CR}}$  relative to the road is

$$v_{\text{CR}} = v_{\text{CT}} + v_{\text{TR}} = 17 \text{ m/s} + 11 \text{ m/s} = \boxed{28 \text{ m/s}}$$

**SSM** **REASONING** The angle  $\theta$  can be found from

$$\theta = \tan^{-1} \left( \frac{2400 \text{ m}}{x} \right) \quad (1)$$

where  $x$  is the horizontal displacement of the flare. Since  $a_x = 0 \text{ m/s}^2$ , it follows that  $x = (v_0 \cos 30.0^\circ)t$ . The flight time  $t$  is determined by the vertical motion. In particular, the time  $t$  can be found from Equation 3.5b. Once the time is known,  $x$  can be calculated.

**SOLUTION** From Equation 3.5b, assuming upward is the positive direction, we have

$$y = -(v_0 \sin 30.0^\circ)t + \frac{1}{2}a_y t^2$$

which can be rearranged to give the following equation that is quadratic in  $t$ :

$$\frac{1}{2}a_y t^2 - (v_0 \sin 30.0^\circ)t - y = 0$$

Using  $y = -2400 \text{ m}$  and  $a_y = -9.80 \text{ m/s}^2$  and suppressing the units, we obtain the quadratic equation

$$4.9t^2 + 120t - 2400 = 0$$

Using the quadratic formula, we obtain  $t = 13 \text{ s}$ . Therefore, we find that

$$x = (v_0 \cos 30.0^\circ)t = (240 \text{ m/s})(\cos 30.0^\circ)(13 \text{ s}) = 2700 \text{ m}$$

Equation (1) then gives

$$\theta = \tan^{-1} \left( \frac{2400 \text{ m}}{2700 \text{ m}} \right) = \boxed{42^\circ}$$

76. **REASONING** The relative velocities in this problem are:

$v_{\text{PW}}$  = velocity of the Passenger relative to the Water

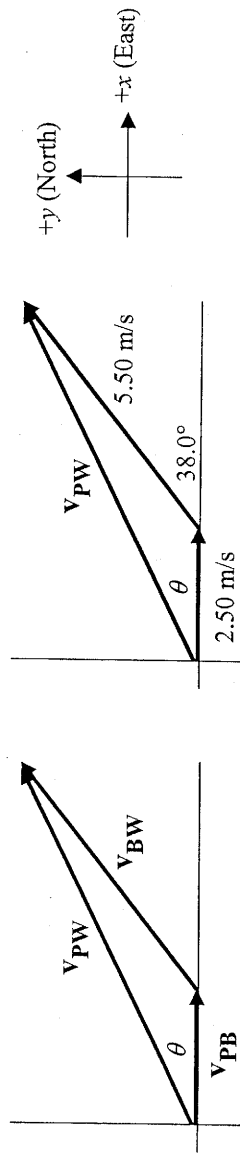
$v_{\text{PB}}$  = velocity of the Passenger relative to the Boat (2.50 m/s, due east)

$v_{\text{BW}}$  = velocity of the Boat relative to the Water (5.50 m/s, at  $38.0^\circ$  north of east)

The velocities are shown in the drawing are related by the subscripting method discussed in Section 3.4:

$$v_{\text{PW}} = v_{\text{PB}} + v_{\text{BW}}$$





We will determine the magnitude and direction of  $v_{PW}$  from the equation above by using the method of scalar components.

**SOLUTION** The table below lists the scalar components of the three vectors.

Vector	x Component	y Component
$v_{PB}$	+2.50 m/s	0 m/s
$v_{BW}$	$+(5.50 \text{ m/s}) \cos 38.0^\circ = +4.33 \text{ m/s}$	$+(5.50 \text{ m/s}) \sin 38.0^\circ = +3.39 \text{ m/s}$
$v_{PW} = v_{PB} + v_{BW}$	$+2.50 \text{ m/s} + 4.33 \text{ m/s} = +6.83 \text{ m/s}$	$0 \text{ m/s} + 3.39 \text{ m/s} = +3.39 \text{ m/s}$

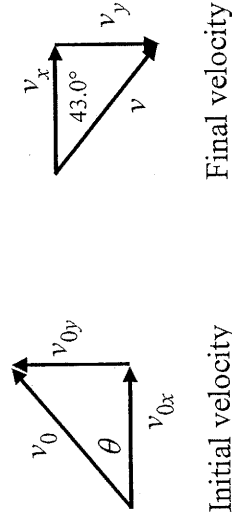
The magnitude of  $v_{PW}$  can be found by applying the Pythagorean theorem to the x and y components:

$$v_{PW} = \sqrt{(6.83 \text{ m/s})^2 + (3.39 \text{ m/s})^2} = \boxed{7.63 \text{ m/s}}$$

The angle  $\theta$  (see the drawings) that  $v_{PW}$  makes with due east is

$$\theta = \tan^{-1} \left( \frac{+3.39 \text{ m/s}}{+6.83 \text{ m/s}} \right) = \boxed{26.4^\circ \text{ north of east}}$$

77. **REASONING** The drawings show the initial and final velocities of the ski jumper and their scalar components. The initial speed of the ski jumper is given by  $v_0 = \sqrt{v_{0x}^2 + v_{0y}^2}$ , and the angle that the initial velocity makes with the horizontal is  $\theta = \tan^{-1} \left( \frac{v_{0y}}{v_{0x}} \right)$ . The scalar components  $v_{0x}$  and  $v_{0y}$  can be determined by using the equations of kinematics and the data in the following tables. (The +x direction is in the direction of the horizontal displacement of the skier, and the +y direction is "up.")



Initial velocity

Final velocity

#### x-Direction Data

x	$a_x$	$v_x$	$v_{0x}$	t
+51.0 m	$0 \text{ m/s}^2$	$+(23.0 \text{ m/s}) \cos 43.0^\circ = +16.8 \text{ m/s}$	?	

#### y-Direction Data

y	$a_y$	$v_y$	$v_{0y}$	t
	$-9.80 \text{ m/s}^2$	$-(23.0 \text{ m/s}) \sin 43.0^\circ = -15.7 \text{ m/s}$	?	

Since there is no acceleration in the x direction ( $a_x = 0 \text{ m/s}^2$ ),  $v_{0x}$  is the same as  $v_x$ , so we have that  $v_{0x} = v_x = +16.8 \text{ m/s}$ . The time that the skier is in the air can be found from the x-direction data, since three of the variables are known. With the value for the time and the y-direction data, the y component of the initial velocity can be determined.

**SOLUTION** Since  $a_x = 0 \text{ m/s}^2$ , the time can be determined from Equation 3.5a as  $t = \frac{x}{v_{0x}} = \frac{+51.0 \text{ m}}{+16.8 \text{ m/s}} = 3.04 \text{ s}$ . The value for  $v_{0y}$  can now be found by using Equation 3.3b with this value of the time and the y-direction data:

$$v_{0y} = v_y - a_y t = -15.7 \text{ m/s} - (-9.80 \text{ m/s}^2)(3.04 \text{ s}) = +14.1 \text{ m/s}$$

The speed of the skier when he leaves the end of the ramp is

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(+16.8 \text{ m/s})^2 + (+14.1 \text{ m/s})^2} = \boxed{21.9 \text{ m/s}}$$

The angle that the initial velocity makes with respect to the horizontal is

$$\theta = \tan^{-1} \left( \frac{v_{0y}}{v_{0x}} \right) = \tan^{-1} \left( \frac{+14.1 \text{ m/s}}{+16.8 \text{ m/s}} \right) = \boxed{40.0^\circ}$$