where h_f is the maximum height of the ball when it rebounds and v_A is the speed of the ball just after it rebounds from the ground. Solving for v_A gives

$$v_A = \sqrt{2gh_f}$$

Substituting equations (2) and (3) into equation (1), where $\mathbf{v_0} = -v_B$, and $\mathbf{v_f} = v_A$ gives (taking "upward" as the positive direction)

$$(\Sigma\overline{\mathbf{F}})\,\Delta t = m\sqrt{2g}\,\left[+\sqrt{h_f} - \left(-\sqrt{h_0}\right)\right]$$

$$= (0.500\text{ kg})\sqrt{2\left(9.80\text{ m/s}^2\right)}\left[+\sqrt{0.700\text{ m}} - \left(-\sqrt{1.20\text{ m}}\right)\right] = \boxed{+4.28\text{ N}\cdot\text{s}}$$

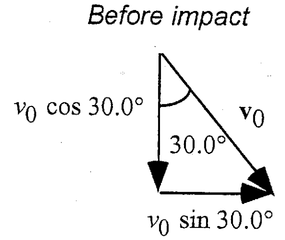
Since the impulse is positive, it is directed $\boxed{\text{upward}}$.

13. SSM WWW ***REASONING*** The impulse applied to the golf ball by the floor can be found from Equation 7.4, the impulse-momentum theorem: $(\Sigma\overline{\mathbf{F}})\,\Delta t = m\mathbf{v}_f - m\mathbf{v}_0$. Two forces act on the golf ball, the average force $\overline{\mathbf{F}}$ exerted by the floor, and the weight of the golf ball. Since $\overline{\mathbf{F}}$ is much greater than the weight of the golf ball, the net average force $(\Sigma\overline{\mathbf{F}})$ is equal to $\overline{\mathbf{F}}$.

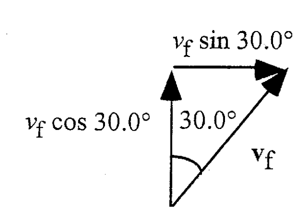
Only the vertical component of the ball's momentum changes during impact with the floor. In order to use Equation 7.4 directly, we must first find the vertical components of the initial and final velocities. We begin, then, by finding these velocity components.

SOLUTION The figures below show the initial and final velocities of the golf ball.

Before impact



After impact



If we take up as the positive direction, then the vertical components of the initial and final velocities are, respectively, $\mathbf{v_{0y}} = -v_0 \cos 30.0°$ and $\mathbf{v_{fy}} = +v_f \cos 30.0°$. Then, from Equation 7.4 the impulse is

$$\overline{\mathbf{F}}\,\Delta t = m(\mathbf{v_{fy}} - \mathbf{v_{0y}}) = m\left[(+v_f \cos 30.0°) - (-v_0 \cos 30.0°)\right]$$

Since $v_0 = v_f = 45\text{ m/s}$, the impulse applied to the golf ball by the floor is

$$\overline{\mathbf{F}}\Delta t = 2mv_0 \cos 30.0° = 2(0.047\text{ kg})(45\text{ m/s})(\cos 30.0°) = \boxed{3.7\text{ N}\cdot\text{s}}$$

REASONING This is a problem in vector addition, and we will use the component method for vector addition. Using this method, we will add the components of the individual momenta in the direction due north to obtain the component of the vector sum in the direction due north. We will obtain the component of the vector sum in the direction due east in a similar fashion from the individual components in that direction. For each jogger the momentum is the mass times the velocity.

SOLUTION Assuming that the directions north and east are positive, the components of the joggers' momenta are as shown in the following table:

	Direction due east	Direction due north
85 kg jogger	$(85\text{ kg})(2.0\text{ m/s}) = 170\text{ kg}\cdot\text{m/s}$	0 kg·m/s
55 kg jogger	$(55\text{ kg})(3.0\text{ m/s})\cos 32° = 140\text{ kg}\cdot\text{m/s}$	$(55\text{ kg})(3.0\text{ m/s})\sin 32° = 87\text{ kg}\cdot\text{m/s}$
Total	310 kg·m /s	87 kg·m/s

Using the Pythagorean theorem, we find that the magnitude of the total momentum is

$$\sqrt{(310\text{ kg}\cdot\text{m/s})^2 + (87\text{ kg}\cdot\text{m/s})^2} = \boxed{322\text{ kg}\cdot\text{m/s}}$$

The total momentum vector points north of east by an angle θ, which is given by

$$\theta = \tan^{-1}\left(\frac{87\text{ kg}\cdot\text{m/s}}{310\text{ kg}\cdot\text{m/s}}\right) = \boxed{16°}$$

15. ***REASONING AND SOLUTION*** The excess weight of the truck is due to the force exerted on the truck by the sand. Newton's third law requires that this force be equal in magnitude to the force exerted on the sand by the truck. In time t, a mass m of sand falls into the truck bed and comes to rest. The impulse is

$$\overline{\mathbf{F}}\Delta t = m(\mathbf{v_f} - \mathbf{v_0}) \quad \text{so} \quad \overline{\mathbf{F}} = \frac{m(\mathbf{v_f} - \mathbf{v_0})}{\Delta t}$$

The sand gains a speed v_0 in falling a height h so

SOLUTION The mass of each piece of the rocket after breakup is m, and so the mass of the rocket before breakup is $2m$. Applying the conservation of momentum theorem along the original line of motion (the x axis) gives

$$\underbrace{mv_1 \cos 30.0° + mv_2 \cos 60.0°}_{P_{f,x}} = \underbrace{2mv_0}_{P_{0,x}} \quad \text{or} \quad v_1 \cos 30.0° + v_2 \cos 60.0° = 2v_0 \quad (1)$$

Applying the conservation of momentum along the y axis gives

$$\underbrace{mv_1 \sin 30.0° - mv_2 \sin 60.0°}_{P_{f,y}} = \underbrace{0}_{P_{0,y}} \quad \text{or} \quad v_2 = \frac{v_1 \sin 30.0°}{\sin 60.0°} \quad (2)$$

a. To find the speed v_1 of the first piece, we substitute the value for v_2 from Equation (2) into Equation (1). The result is

$$v_1 \cos 30.0° + \left(\frac{v_1 \sin 30.0°}{\sin 60.0°}\right) \cos 60.0° = 2v_0$$

Solving for v_1 and setting $v_0 = 45.0$ m/s yields

$$v_1 = \frac{2v_0}{\cos 30.0° + \left(\frac{\sin 30.0°}{\sin 60.0°}\right)\cos 60.0°} = \frac{2(45.0 \text{ m/s})}{\cos 30.0° + \left(\frac{\sin 30.0°}{\sin 60.0°}\right)\cos 60.0°} = \boxed{77.9 \text{ m/s}}$$

b. The speed v_2 of the second piece can be found by substituting $v_1 = 77.9$ m/s into Equation (2):

$$v_2 = \frac{v_1 \sin 30.0°}{\sin 60.0°} = \frac{(77.9 \text{ m/s}) \sin 30.0°}{\sin 60.0°} = \boxed{45.0 \text{ m/s}}$$

---

23. $\boxed{\text{SSM}}$ ***REASONING*** No net external force acts on the plate parallel to the floor; therefore, the component of the momentum of the plate that is parallel to the floor is conserved as the plate breaks and flies apart. Initially, the total momentum parallel to the floor is zero. After the collision with the floor, the component of the total momentum parallel to the floor must remain zero. The drawing in the text shows the pieces in the plane parallel to the floor just after the collision. Clearly, the linear momentum in the plane parallel to the floor has two components; therefore the linear momentum of the plate must be conserved in each of these two mutually perpendicular directions. Using the drawing in the text, with the positive directions taken to be up and to the right, we have

$$[x \text{ direction}] \qquad -m_1 v_1 (\sin 25.0°) + m_2 v_2 (\cos 45.0°) = 0 \qquad (1)$$

$$[y \text{ direction}] \qquad m_1 v_1 (\cos 25.0°) + m_2 v_2 (\sin 45.0°) - m_3 v_3 = 0 \qquad (2)$$

These equations can be solved simultaneously for the masses m_1 and m_2.

SOLUTION Using the values given in the drawing for the velocities after the plate breaks, we have,

$$-m_1 (3.00 \text{ m/s}) \sin 25.0° + m_2 (1.79 \text{ m/s}) \cos 45.0° = 0 \qquad (1)$$

$$m_1 (3.00 \text{ m/s}) \cos 25.0° + m_2 (1.79 \text{ m/s}) \sin 45.0° - (1.30 \text{ kg})(3.07 \text{ m/s}) = 0 \qquad (2)$$

Subtracting (2) from (1), and noting that $\cos 45.0° = \sin 45.0°$, gives $\boxed{m_1 = 1.00 \text{ kg}}$.

Substituting this value into either (1) or (2) then yields $\boxed{m_2 = 1.00 \text{ kg}}$.

---

24. ***REASONING*** We will divide the problem into two parts: (a) the motion of the freely falling block after it is dropped from the building and before it collides with the bullet, and (b) the collision of the block with the bullet.

During the falling phase we will use an equation of kinematics that describes the velocity of the block as a function of time (which is unknown). During the collision with the bullet, the external force of gravity acts on the system. This force changes the momentum of the system by a negligibly small amount since the collision occurs over an extremely short time interval. Thus, to a good approximation, the sum of the external forces acting on the system during the collision is negligible, so the linear momentum of the system is conserved. The principle of conservation of linear momentum can be used to provide a relation between the momenta of the system before and after the collision. This relation will enable us to find a value for the time it takes for the bullet/block to reach the top of the building.

SOLUTION Falling from rest ($v_{0,\text{ block}} = 0$ m/s), the block attains a final velocity v_{block} just before colliding with the bullet. This velocity is given by Equation 2.4 as

$$\underbrace{v_{\text{block}}}_{\substack{\text{Final velocity of} \\ \text{block just before} \\ \text{bullet hits it}}} = \underbrace{v_{0,\text{ block}}}_{\substack{\text{Initial velocity of} \\ \text{block at top of} \\ \text{building}}} + at$$

where a is the acceleration due to gravity ($a = -9.8$ m/s^2) and t is the time of fall. The upward direction is assumed to be positive. Therefore, the final velocity of the falling block is

$$v_{\text{block}} = at \qquad (1)$$

During the collision with the bullet, the total linear momentum of the bullet/block system is conserved, so we have that

$$\underbrace{\left(m_{\text{bullet}} + m_{\text{block}}\right)v_{\text{f}}}_{\substack{\text{Total linear momentum}\\ \text{after collision}}} = \underbrace{m_{\text{bullet}}v_{\text{bullet}} + m_{\text{block}}v_{\text{block}}}_{\substack{\text{Total linear momentum}\\ \text{before collision}}} \quad (2)$$

Here v_{f} is the final velocity of the bullet/block system after the collision, and v_{bullet} and v_{block} are the initial velocities of the bullet and block just before the collision. We note that the bullet/block system reverses direction, rises, and comes to a momentary halt at the top of the building. This means that v_{f}, the final velocity of the bullet/block system after the collision must have the same magnitude as v_{block}, the velocity of the falling block just before the bullet hits it. Since the two velocities have opposite directions, it follows that $v_{\text{f}} = -v_{\text{block}}$. Substituting this relation and Equation (1) into Equation (2) gives

$$\left(m_{\text{bullet}} + m_{\text{block}}\right)\left(-at\right) = m_{\text{bullet}}v_{\text{bullet}} + m_{\text{block}}\left(at\right)$$

Solving for the time, we find that

$$t = \frac{-m_{\text{bullet}}v_{\text{bullet}}}{a\left(m_{\text{bullet}} + 2m_{\text{block}}\right)} = \frac{-(0.015\text{ kg})(+810\text{ m/s})}{\left(-9.80\text{ m/s}^2\right)\left[(0.015\text{ kg}) + 2(1.8\text{ kg})\right]} = \boxed{0.34\text{ s}}$$

---

25. ***REASONING*** During the time that the skaters are pushing against each other, the sum of the external forces acting on the two-skater system is zero, because the weight of each skater is balanced by a corresponding normal force and friction is negligible. The skaters constitute an isolated system, so the principle of conservation of linear momentum applies. We will use this principle to find an expression for the ratio of the skater's masses in terms of their recoil velocities. We will then obtain expressions for the recoil velocities by noting that each skater, after pushing off, comes to rest in a certain distance. The recoil velocity, acceleration, and distance are related by Equation 2.9 of the equations of kinematics.

SOLUTION While the skaters are pushing against each other, the total linear momentum of the two-skater system is conserved:

$$\underbrace{m_1 v_{\text{f1}} + m_2 v_{\text{f2}}}_{\substack{\text{Total momentum}\\ \text{after pushing}}} = \underbrace{0}_{\substack{\text{Total momentum}\\ \text{before pushing}}}$$

Solving this expression for the ratio of the masses gives

$$\frac{m_1}{m_2} = -\frac{v_{\text{f2}}}{v_{\text{f1}}} \quad (1)$$

For each skater the (initial) recoil velocity v_{f}, final velocity v, acceleration a, and displacement x are related by Equation 2.9 of the equations of kinematics: $v^2 = v_{\text{f}}^2 + 2ax$. Solving for the recoil velocity gives $v_{\text{f}} = \pm\sqrt{v^2 - 2ax}$. If we assume that skater 1 recoils in the positive direction and skater 2 recoils in the negative direction, the recoil velocities are

Skater 1 $\quad v_{\text{f1}} = +\sqrt{v_1^2 - 2a_1 x_1}$

Skater 2 $\quad v_{\text{f2}} = -\sqrt{v_2^2 - 2a_2 x_2}$

Substituting these expressions into Equation (1) gives

$$\frac{m_1}{m_2} = -\frac{-\sqrt{v_2^2 - 2a_2 x_2}}{\sqrt{v_1^2 - 2a_1 x_1}} = \frac{\sqrt{v_2^2 - 2a_2 x_2}}{\sqrt{v_1^2 - 2a_1 x_1}} \quad (2)$$

Since the skaters come to rest, their final velocities are zero, so $v_1 = v_2 = 0$ m/s. We also know that their accelerations have the same magnitudes. This means that $a_2 = -a_1$, where the minus sign denotes that the acceleration of skater 2 is opposite that of skater 1, since they are moving in opposite directions and are both slowing down. Finally, we are given that skater 1 glides twice as far as skater 2. Thus, the displacement of skater 1 is related to that of skater 2 by $x_1 = -2x_2$, where, the minus sign denotes that the skaters move in opposite directions. Substituting these values into Equation (2) yields

$$\frac{m_1}{m_2} = \frac{\sqrt{(0\text{ m/s})^2 - 2(-a_1)(x_2)}}{\sqrt{(0\text{ m/s})^2 - 2a_1(-2x_2)}} = \frac{1}{\sqrt{2}} = \boxed{0.707}$$

---

26. ***REASONING AND SOLUTION*** Since no net external force acts in the horizontal direction, the total horizontal momentum of the system is conserved regardless of which direction the mass is thrown. The momentum of the system before the mass is thrown off the wagon is $m_W v_A$, where m_W and v_A are the mass and velocity of the wagon, respectively. When ten percent of the wagon's mass is thrown forward, the wagon is brought to a halt so its final momentum is zero. The momentum of the mass thrown forward is $0.1m_W(v_A + v_M)$, where v_M is the velocity of the mass relative to the wagon and $(v_A + v_M)$ is the velocity of the mass relative to the ground. The conservation of momentum gives

34. ***REASONING*** The net external force acting on the two-puck system is zero (the weight of each ball is balanced by an upward normal force, and we are ignoring friction due to the layer of air on the hockey table). Therefore, the two pucks constitute an isolated system, and the principle of conservation of linear momentum applies.

SOLUTION Conservation of linear momentum requires that the total momentum is the same before and after the collision. Since linear momentum is a vector, the x and y components must be conserved separately. Using the drawing in the text, momentum conservation in the x direction yields

$$m_A v_{0A} = m_A v_{fA}\left(\cos 65°\right) + m_B v_{fB}\left(\cos 37°\right) \quad (1)$$

while momentum conservation in the y direction yields

$$0 = m_A v_{fA}\left(\sin 65°\right) - m_B v_{fB}\left(\sin 37°\right) \quad (2)$$

Solving equation (2) for v_{fB}, we find that

$$v_{fB} = \frac{m_A v_{fA}\left(\sin 65°\right)}{m_B\left(\sin 37°\right)} \quad (3)$$

Substituting equation (3) into Equation (1) leads to

$$m_A v_{0A} = m_A v_{fA}\left(\cos 65°\right) + \left[\frac{m_A v_{fA}\left(\sin 65°\right)}{\sin 37°}\right]\left(\cos 37°\right)$$

a. Solving for v_{fA} gives

$$v_{fA} = \frac{v_{0A}}{\cos 65° + \left(\dfrac{\sin 65°}{\tan 37°}\right)} = \frac{+5.5 \text{ m/s}}{\cos 65° + \left(\dfrac{\sin 65°}{\tan 37°}\right)} = \boxed{3.4 \text{ m/s}}$$

b. From equation (3), we find that

$$v_{fB} = \frac{(0.025 \text{ kg})(3.4 \text{ m/s})(\sin 65°)}{(0.050 \text{ kg})(\sin 37°)} = \boxed{2.6 \text{ m/s}}$$

35. SSM ***REASONING*** Batman and the boat with the criminal constitute the system. Gravity acts on this system as an external force; however, gravity acts vertically, and we are concerned only with the horizontal motion of the system. If we neglect air resistance and friction, there are no external forces that act horizontally; therefore, the total linear momentum in the horizontal direction is conserved. When Batman collides with the boat, the horizontal component of his velocity is zero, so the statement of conservation of linear momentum in the horizontal direction can be written as

$$\underbrace{(m_1 + m_2)v_f}_{\substack{\text{Total horizontal momentum}\\\text{after collision}}} = \underbrace{m_1 v_{01} + 0}_{\substack{\text{Total horizontal momentum}\\\text{before collision}}}$$

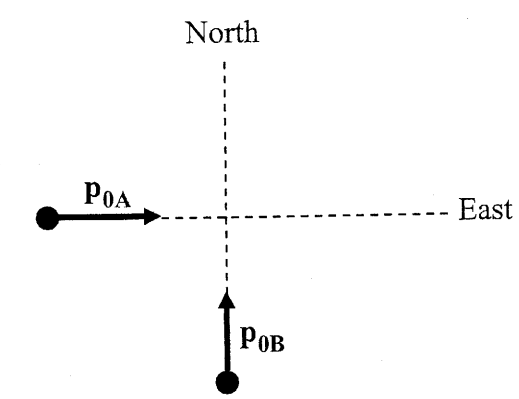
Here, m_1 is the mass of the boat, and m_2 is the mass of Batman. This expression can be solved for v_f, the velocity of the boat after Batman lands in it.

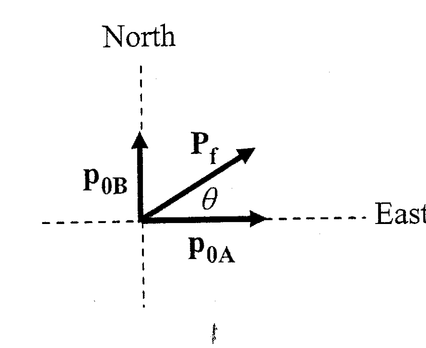
SOLUTION Solving for v_f gives

$$v_f = \frac{m_1 v_{01}}{m_1 + m_2} = \frac{(510 \text{ kg})(+11 \text{ m/s})}{510 \text{ kg} + 91 \text{ kg}} = \boxed{+9.3 \text{ m/s}}$$

The plus sign indicates that the boat continues to move in its initial direction of motion.

36. ***REASONING*** According to the momentum-conservation principle, the final total momentum is the same as the initial total momentum. The initial total momentum is the vector sum of the two initial momentum vectors of the objects. One of the vectors ($\mathbf{p_{0A}}$) points due east and one ($\mathbf{p_{0B}}$) due north, so that they are perpendicular (see the drawing on the left).





Based on the conservation principle, the direction of the final total momentum $\mathbf{P_f}$ must be the same as the direction of the initial total momentum. From the drawing on the right, we can see that the initial total momentum has a component $\mathbf{p_{0A}}$ pointing due east and a component $\mathbf{p_{0B}}$ pointing due north. The final total momentum $\mathbf{P_f}$ has these same components and, therefore, must point north of east at an angle θ.

SOLUTION Based on the conservation of linear momentum, we know that the magnitude of the final total momentum is the same as the magnitude of the initial total momentum. Using the Pythagorean theorem with the initial momentum vectors of the two objects, we find that the magnitude of the final total momentum is

$$P_f = \sqrt{p_{0A}^2 + p_{0B}^2} = \sqrt{(m_A v_{0A})^2 + (m_B v_{0B})^2}$$

$$P_f = \sqrt{(17.0\text{ kg})^2(8.00\text{ m/s})^2 + (29.0\text{ kg})^2(5.00\text{ m/s})^2} = \boxed{199\text{ kg}\cdot\text{m/s}}$$

The direction is given by

$$\theta = \tan^{-1}\left(\frac{p_{0B}}{p_{0A}}\right) = \tan^{-1}\left(\frac{m_B v_{0B}}{m_A v_{0A}}\right) = \tan^{-1}\left[\frac{(29.0\text{ kg})(5.00\text{ m/s})}{(17.0\text{ kg})(8.00\text{ m/s})}\right] = \boxed{46.8^\circ\text{ north of east}}$$

---

37. ***REASONING*** The velocity of the second ball just after the collision can be found from Equation 7.8b (see Example 7). In order to use Equation 7.8b, however, we must know the velocity of the first ball just before it strikes the second ball. Since we know the impulse delivered to the first ball by the pool stick, we can use the impulse-momentum theorem (Equation 7.4) to find the velocity of the first ball just before the collision.

SOLUTION According to the impulse-momentum theorem, $\overline{F}\Delta t = mv_f - mv_0$, and setting $v_0 = 0$ m/s and solving for v_f, we find that the velocity of the first ball after it is struck by the pool stick and just before it hits the second ball is

$$v_f = \frac{\overline{F}\Delta t}{m} = \frac{+1.50\text{ N}\cdot\text{s}}{0.165\text{ kg}} = 9.09\text{ m/s}$$

Substituting values into Equation 7.8b (with $v_{01} = 9.09$ m/s), we have

$$v_{f2} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{01} = \left(\frac{2m}{m+m}\right)v_{01} = v_{01} = \boxed{+9.09\text{ m/s}}$$

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38. ***REASONING***

a. Since air resistance is negligible as the ball swings downward, the work done by nonconservative forces is zero, $W_{nc} = 0$ J. The force due to the tension in the wire is perpendicular to the motion and, therefore, does no work. Thus, the total mechanical energy, which is the sum of the kinetic and potential energies, is conserved (see Section 6.5). The conservation of mechanical energy will be used to find the speed of the ball just before it collides with the block.

b. When the ball collides with the stationary block, the collision is elastic. This means that, during the collision, the total kinetic energy of the system is conserved. The horizontal or x-component of the total momentum is conserved, because the horizontal surface is frictionless, and so the net average external force acting on the ball-block system in the horizontal direction is zero. The total kinetic energy is conserved, since the collision is known to be elastic.

SOLUTION

a. As the ball falls, the total mechanical energy is conserved. Thus, the total mechanical energy at the top of the swing is equal to that at the bottom:

$$\underbrace{m_1 g L}_{\substack{\text{Total mechanical}\\\text{energy at top of}\\\text{swing, all}\\\text{potential energy}}} = \underbrace{\tfrac{1}{2}m_1 v^2}_{\substack{\text{Total mechanical}\\\text{energy at bottom}\\\text{of swing, all}\\\text{kinetic energy}}}$$

In this expression L is the length of the wire, m_1 is the mass of the ball, g is the magnitude of the acceleration due to gravity, and v is the speed of the ball just before the collision. We have chosen the zero-level for the potential energy to be at ground level, so the initial potential energy of the ball is $m_1 gL$. Solving for the speed v of the ball gives

$$v = \sqrt{2gL} = \sqrt{2(9.80\text{ m/s}^2)(1.20\text{ m})} = 4.85\text{ m/s}$$

At the bottom of the swing the ball is moving horizontally and to the right, which we take to be the $+x$ direction (see the drawing in the text). Thus the velocity of the ball just before impact is $v_x = \boxed{+4.85\text{ m/s}}$.

b. The conservation of linear momentum and the conservation of the total kinetic energy can be used to describe the behavior of the system during the elastic collision. This situation is identical to that in Example 7 in Section 7.3, so Equation 7.8a applies:

$$v_f = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_x$$

where v_f is the final velocity of the ball after the collision, m_1 and m_2 are, respectively, the masses of the ball and block, and v_x is the velocity of the ball just before the collision. Since $v_x = +4.85$ m/s, we find that

$$v_f = \left(\frac{1.60\text{ kg} - 2.40\text{ kg}}{1.60\text{ kg} + 2.40\text{ kg}}\right)(+4.85\text{ m/s}) = \boxed{-0.97\text{ m/s}}$$

The minus sign indicates that the ball rebounds to the left after the collision.

---

39. ***REASONING AND SOLUTION*** The total linear momentum of the two-car system is conserved because no net external force acts on the system during the collision. We are ignoring friction *during the collision*, and the weights of the cars are balanced by the normal forces exerted by the ground. Momentum conservation gives

$$\underbrace{(m_1 + m_2)v_f}_{\substack{\text{Total momentum}\\\text{after collision}}} = \underbrace{m_1 v_{01} + m_2 v_{02}}_{\substack{\text{Total momentum}\\\text{after collision}}}$$

where $v_{02} = 0$ m/s since the 1900-kg car is stationary before the collision.

a. Solving for v_f, we find that the velocity of the two cars just after the collision is

$$v_f = \frac{m_1 v_{01} + m_2 v_{02}}{m_1 + m_2} = \frac{(2100\text{ kg})(+17\text{ m/s}) + (1900\text{ kg})(0\text{ m/s})}{2100\text{ kg} + 1900\text{ kg}} = \boxed{+8.9\text{ m/s}}$$

The plus sign indicates that the velocity of the two cars just after the collision is in the same direction as the direction of the velocity of the 2100-kg car before the collision.

b. According to the impulse-momentum theorem, Equation 7.4, we have

$$\underbrace{\overline{F}\Delta t}_{\substack{\text{Impulse}\\\text{due to}\\\text{friction}}} = \underbrace{(m_1 + m_2)v_{\text{final}}}_{\substack{\text{Final momentum}\\\text{when cars come}\\\text{to a halt}}} - \underbrace{(m_1 + m_2)v_{\text{after}}}_{\substack{\text{Total momentum}\\\text{just after collision}}}$$

where $v_{\text{final}} = 0$ m/s since the cars come to a halt, and $v_{\text{after}} = v_f = +8.9\text{ m/s}$. Therefore, we have

$$\overline{F}\Delta t = (2100\text{ kg} + 1900\text{ kg})(0\text{ m/s}) - (2100\text{ kg} + 1900\text{ kg})(8.9\text{ m/s}) = \boxed{-3.6 \times 10^4\text{ N}\cdot\text{s}}$$

The minus sign indicates that the impulse due to friction acts opposite to the direction of motion of the locked, two-car system. This is reasonable since the velocity of the cars is decreasing in magnitude as the cars skid to a halt.

c. Using the same notation as in part (b) above, we have from the equations of kinematics (Equation 2.9) that

$$v_{\text{final}}^2 = v_{\text{after}}^2 + 2ax$$

where $v_{\text{final}} = 0$ m/s and $v_{\text{after}} = v_f = +8.9\text{ m/s}$. From Newton's second law we have that $a = -f_k/(m_1 + m_2)$, where f_k is the force of kinetic friction that acts on the cars as they skid to a halt. Therefore,

$$0 = v_f^2 + 2\left(\frac{-f_k}{m_1 + m_2}\right)x \quad \text{or} \quad x = \frac{(m_1 + m_2)v_f^2}{2f_k}$$

According to Equation 4.8, $f_k = \mu_k F_N$, where μ_k is the coefficient of kinetic friction and F_N is the magnitude of the normal force that acts on the two-car system. There are only two vertical forces that act on the system; they are the upward normal force $\mathbf{F}_N$ and the weight $(m_1 + m_2)\mathbf{g}$ of the cars. Taking upward as the positive direction and applying Newton's second law in the vertical direction, we have $F_N - (m_1 + m_2)g = (m_1 + m_2)a_y = 0$, or $F_N = (m_1 + m_2)g$. Therefore, $f_k = \mu_k(m_1 + m_2)g$, and we have

$$x = \frac{(m_1 + m_2)v_f^2}{2\mu_k(m_1 + m_2)g} = \frac{v_f^2}{2\mu_k g} = \frac{(8.9\text{ m/s})^2}{2(0.68)(9.80\text{ m/s}^2)} = \boxed{5.9\text{ m}}$$

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40. ***REASONING*** Considering the boat and the stone as a single system, there is no net external horizontal force acting on the stone-boat system, and thus the horizontal component of the system's linear momentum is conserved: $m_1 v_{f1x} + m_2 v_{f2x} = m_1 v_{01x} + m_2 v_{02x}$ (Equation 7.9a). We have taken east as the positive direction.

SOLUTION We will use the following symbols in solving the problem:

m_1 = mass of the stone = 0.072 kg
v_{f1} = final speed of the stone = 11 m/s
v_{01} = initial speed of the stone = 13 m/s
m_2 = mass of the boat
v_{f2} = final speed of the boat = 2.1 m/s
v_{02} = initial speed of the boat = 0 m/s

Because the boat is initially at rest, v_{02x} = 0 m/s, and Equation 7.9a reduces to $m_1 v_{f1x} + m_2 v_{f2x} = m_1 v_{01x}$. Solving for the mass m_2 of the boat, we obtain

$$m_2 v_{f2x} = m_1 v_{01x} - m_1 v_{f1x} \quad \text{or} \quad m_2 = \frac{m_1(v_{01x} - v_{f1x})}{v_{f2x}}$$