SOLUTION The average angular acceleration is given by Equation 8.4 as $\bar{\alpha} = \frac{\omega - \omega_0}{t - t_0}$ where $t - t_0 = 4.0$ s is the elapsed time.

(a) $$\bar{\alpha} = \frac{\omega - \omega_0}{t - t_0} = \frac{+5.0\text{ rad/s} - 2.0\text{ rad/s}}{4.0\text{ s}} = \boxed{+0.75\text{ rad/s}^2}$$

(b) $$\bar{\alpha} = \frac{\omega - \omega_0}{t - t_0} = \frac{+2.0\text{ rad/s} - 5.0\text{ rad/s}}{4.0\text{ s}} = \boxed{-0.75\text{ rad/s}^2}$$

(c) $$\bar{\alpha} = \frac{\omega - \omega_0}{t - t_0} = \frac{-3.0\text{ rad/s} - (-7.0\text{ rad/s})}{4.0\text{ s}} = \boxed{+1.0\text{ rad/s}^2}$$

(d) $$\bar{\alpha} = \frac{\omega - \omega_0}{t - t_0} = \frac{-4.0\text{ rad/s} - (+4.0\text{ rad/s})}{4.0\text{ s}} = \boxed{-2.0\text{ rad/s}^2}$$

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8. ***REASONING*** The relation between the final angular velocity ω, the initial angular velocity ω_0, and the angular acceleration α is given by Equation 8.4 (with $t_0 = 0$ s) as

$$\omega = \omega_0 + \alpha t$$

If α has the same sign as ω_0, then the angular speed, which is the magnitude of the angular velocity ω, is increasing. On the other hand, If α and ω_0 have opposite signs, then the angular speed is decreasing.

SOLUTION According to Equation 8.4, we know that $\omega = \omega_0 + \alpha t$. Therefore, we find:

(a) $\omega = +12\text{ rad/s} + (+3.0\text{ rad/s}^2)(2.0\text{ s}) = +18\text{ rad/s}$. The angular speed is $\boxed{18\text{ rad/s}}$.

(b) $\omega = +12\text{ rad/s} + (-3.0\text{ rad/s}^2)(2.0\text{ s}) = +6.0\text{ rad/s}$. The angular speed is $\boxed{6.0\text{ rad/s}}$.

(c) $\omega = -12\text{ rad/s} + (+3.0\text{ rad/s}^2)(2.0\text{ s}) = -6.0\text{ rad/s}$. The angular speed is $\boxed{6.0\text{ rad/s}}$.

(d) $\omega = -12\text{ rad/s} + (-3.0\text{ rad/s}^2)(2.0\text{ s}) = -18\text{ rad/s}$. The angular speed is $\boxed{18\text{ rad/s}}$.

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9. SSM ***REASONING*** Equation 8.4 $\left[\bar{\alpha} = (\omega - \omega_0)/t\right]$ indicates that the average angular acceleration is equal to the change in the angular velocity divided by the elapsed time. Since the wheel starts from rest, its initial angular velocity is $\omega_0 = 0$ rad/s. Its final angular velocity is given as $\omega = 0.24$ rad/s. Since the average angular acceleration is given as $\bar{\alpha} = 0.030\text{ rad/s}^2$, Equation 8.4 can be solved to determine the elapsed time t.

SOLUTION Solving Equation 8.4 for the elapsed time gives

$$t = \frac{\omega - \omega_0}{\bar{\alpha}} = \frac{0.24\text{ rad/s} - 0\text{ rad/s}}{0.030\text{ rad/s}^2} = \boxed{8.0\text{ s}}$$

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10. ***REASONING AND SOLUTION*** Using Equation 8.4 and the appropriate conversion factors, the average angular acceleration of the CD in rad/s² is

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \left(\frac{210\text{ rev/min} - 480\text{ rev/min}}{74\text{ min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)^2 = -6.4\times10^{-3}\text{ rad/s}^2$$

The magnitude of the average angular acceleration is $\boxed{6.4\times10^{-3}\text{ rad/s}^2}$.

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11. ***REASONING*** The average angular velocity $\bar{\omega}$ is defined as the angular displacement $\Delta\theta$ divided by the elapsed time Δt during which the displacement occurs: $\bar{\omega} = \Delta\theta/\Delta t$ (Equation 8.2). Solving for the elapsed time gives $\Delta t = \Delta\theta/\bar{\omega}$. We are given $\Delta\theta$ and can calculate $\bar{\omega}$ from the fact that the earth rotates on its axis once every 24.0 hours.

SOLUTION The sun itself subtends an angle of 9.28×10^{-3} rad. When the sun moves a distance equal to its diameter, it moves through an angle that is also 9.28×10^{-3} rad; thus, $\Delta\theta = 9.28\times10^{-3}$ rad. The average angular velocity $\bar{\omega}$ at which the sun appears to move across the sky is the same as that of the earth rotating on its axis, $\bar{\omega}_{\text{earth}}$, so $\bar{\omega} = \bar{\omega}_{\text{earth}}$. Since the earth makes one revolution (2π rad) every 24.0 h, its average angular velocity is

$$\bar{\omega}_{\text{earth}} = \frac{\Delta\theta_{\text{earth}}}{\Delta t_{\text{earth}}} = \frac{2\pi\text{ rad}}{24.0\text{ h}} = \frac{2\pi\text{ rad}}{(24.0\cancel{\text{h}})\left(\frac{3600\text{ s}}{1\cancel{\text{h}}}\right)} = 7.27\times10^{-5}\text{ rad/s}$$

The time it takes for the sun to move a distance equal to its diameter is

$$\Delta t = \frac{\Delta\theta}{\bar{\omega}_{\text{earth}}} = \frac{9.28\times10^{-3}\text{ rad}}{7.27\times10^{-5}\text{ rad/s}} = \boxed{128\text{ s (a little over 2 minutes)}}$$

12. ***REASONING AND SOLUTION*** The angular displacements of the astronauts are equal.

For A $\qquad \theta = s_A/r_A$

For B $\qquad \theta = s_B/r_B$

Equating these two equations for θ and solving for s_B gives

$$s_B = (r_B/r_A)s_A = [(1.10 \times 10^3 \text{ m})/(3.20 \times 10^2 \text{ m})](2.40 \times 10^2 \text{ m}) = \boxed{825 \text{ m}}$$

13. ***REASONING AND SOLUTION*** The people meet at time t. At this time the magnitude of their angular displacements must total 2π rad.

$$\theta_1 + \theta_2 = 2\pi \text{ rad}$$

Then

$$\omega_1 t + \omega_2 t = 2\pi \text{ rad}$$

$$t = \frac{2\pi \text{ rad}}{\omega_1 + \omega_2} = \frac{2\pi \text{ rad}}{1.7 \times 10^{-3} \text{ rad/s} + 3.4 \times 10^{-3} \text{ rad/s}} = \boxed{1200 \text{ s}}$$

14. ***REASONING*** It does not matter whether the arrow is aimed closer to or farther away from the axis. The blade edge sweeps through the open angular space as a rigid unit. This means that a point closer to the axis has a smaller distance to travel along the circular arc in order to bridge the angular opening and correspondingly has a smaller tangential speed. A point farther from the axis has a greater distance to travel along the circular arc but correspondingly has a greater tangential speed. These speeds have just the right values so that all points on the blade edge bridge the angular opening in the same time interval.

The rotational speed of the blades must not be so fast that one blade rotates into the open angular space while part of the arrow is still there. A faster arrow speed means that the arrow spends less time in the open space. Thus, the blades can rotate more quickly into the open space without hitting the arrow, so the maximum value of the angular speed ω increases with increasing arrow speed v.

A longer arrow traveling at a given speed means that some part of the arrow is in the open space for a longer time. To avoid hitting the arrow, then, the blades must rotate more slowly. Thus, the maximum value of the angular speed ω decreases with increasing arrow length L.

The time during which some part of the arrow remains in the open angular space is t_{Arrow}. The time it takes for the edge of a propeller blade to rotate through the open angular space between the blades is t_{Blade}. The maximum angular speed is the angular speed such that these two times are equal.

SOLUTION The time during which some part of the arrow remains in the open angular space is the time it takes the arrow to travel at a speed v through a distance equal to its own length L. This time is $t_{\text{Arrow}} = L/v$. The time it takes for the edge to rotate at an angular speed ω through the angle θ between the blades is $t_{\text{Blade}} = \theta/\omega$. The maximum angular speed is the angular speed such that these two times are equal. Therefore, we have

$$\underbrace{\frac{L}{v}}_{\text{Arrow}} = \underbrace{\frac{\theta}{\omega}}_{\text{Blade}}$$

In this expression we note that the value of the angular opening is $\theta = 60.0°$, which is $\theta = \frac{1}{6}(2\pi) \text{ rad} = \frac{1}{3}\pi \text{ rad}$. Solving the expression for ω gives

$$\omega = \frac{\theta v}{L} = \frac{\pi v}{3L}$$

Substituting the given values for v and L into this result, we find that

a. $$\omega = \frac{\pi v}{3L} = \frac{\pi(75.0 \text{ m/s})}{3(0.71 \text{ m})} = \boxed{111 \text{ rad/s}}$$

b. $$\omega = \frac{\pi v}{3L} = \frac{\pi(91.0 \text{ m/s})}{3(0.71 \text{ m})} = \boxed{134 \text{ rad/s}}$$

c. $$\omega = \frac{\pi v}{3L} = \frac{\pi(91.0 \text{ m/s})}{3(0.81 \text{ m})} = \boxed{118 \text{ rad/s}}$$

15. ***REASONING AND SOLUTION*** The baton will make four revolutions in a time t given by

$$t = \frac{\theta}{\omega}$$

Half of this time is required for the baton to reach its highest point. The magnitude of the initial vertical velocity of the baton is then

24. ***REASONING*** The angular displacement is given by Equation 8.6 as the product of the average angular velocity and the time

$$\theta = \bar{\omega} t = \underbrace{\tfrac{1}{2}(\omega_0 + \omega)}_{\text{Average angular velocity}} t$$

This value for the angular displacement is greater than $\omega_0 t$. When the angular displacement θ is given by the expression $\theta = \omega_0 t$, it is assumed that the angular velocity remains constant at its initial (and smallest) value of ω_0 for the entire time, which does not, however, account for the additional angular displacement that occurs because the angular velocity is increasing.

The angular displacement is also less than ωt. When the angular displacement is given by the expression $\theta = \omega t$, it is assumed that the angular velocity remains constant at its final (and largest) value of ω for the entire time, which does not account for the fact that the wheel was rotating at a smaller angular velocity during the time interval.

SOLUTION
a. If the angular velocity is constant and equals the initial angular velocity ω_0, then $\bar{\omega} = \omega_0$ and the angular displacement is

$$\theta = \omega_0 t = (+220 \text{ rad/s})(10.0 \text{ s}) = \boxed{+2200 \text{ rad}}$$

b. If the angular velocity is constant and equals the final angular velocity ω, then $\bar{\omega} = \omega$ and the angular displacement is

$$\theta = \omega t = (+280 \text{ rad/s})(10.0 \text{ s}) = \boxed{+2800 \text{ rad}}$$

c. Using the definition of average angular velocity, we have

$$\theta = \tfrac{1}{2}(\omega_0 + \omega)t = \tfrac{1}{2}(+220 \text{ rad/s} + 280 \text{ rad/s})(10.0 \text{ s}) = \boxed{+2500 \text{ rad}} \qquad (8.6)$$

25. ***REASONING***
a. The time t for the wheels to come to a halt depends on the initial and final velocities, ω_0 and ω, and the angular displacement θ: $\theta = \tfrac{1}{2}(\omega_0 + \omega)t$ (see Equation 8.6). Solving for the time yields

$$t = \frac{2\theta}{\omega_0 + \omega}$$

b. The angular acceleration α is defined as the change in the angular velocity, $\omega - \omega_0$, divided by the time t:

$$\alpha = \frac{\omega - \omega_0}{t} \qquad (8.4)$$

SOLUTION
a. Since the wheel comes to a rest, $\omega = 0$ rad/s. Converting 15.92 revolutions to radians (1 rev = 2π rad), the time for the wheel to come to rest is

$$t = \frac{2\theta}{\omega_0 + \omega} = \frac{2(+15.92 \cancel{\text{rev}})\left(\dfrac{2\pi \text{ rad}}{1 \cancel{\text{rev}}}\right)}{+20.0 \text{ rad/s} + 0 \text{ rad/s}} = \boxed{10.0 \text{ s}}$$

b. The angular acceleration is

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 \text{ rad/s} - 20.0 \text{ rad/s}}{10.0 \text{ s}} = \boxed{-2.00 \text{ rad/s}^2}$$

26. ***REASONING*** Equation 8.8 $\left(\omega^2 = \omega_0^2 + 2\alpha\theta\right)$ from the equations of rotational kinematics can be employed to find the final angular velocity ω. The initial angular velocity is $\omega_0 = 0$ rad/s since the top is initially at rest, and the angular acceleration is given as $\alpha = 12$ rad/s^2. The angle θ (in radians) through which the pulley rotates is not given, but it can be obtained from Equation 8.1 ($\theta = s/r$), where the arc length s is the 64-cm length of the string and r is the 2.0-cm radius of the top.

SOLUTION Solving Equation 8.8 for the final angular velocity gives

$$\omega = \pm\sqrt{\omega_0^2 + 2\alpha\theta}$$

We choose the positive root, because the angular acceleration is given as positive and the top is at rest initially. Substituting $\theta = s/r$ from Equation 8.1 gives

$$\omega = +\sqrt{\omega_0^2 + 2\alpha\left(\frac{s}{r}\right)} = +\sqrt{(0 \text{ rad/s})^2 + 2(12 \text{ rad/s}^2)\left(\frac{64 \text{ cm}}{2.0 \text{ cm}}\right)} = \boxed{28 \text{ rad/s}}$$

27. SSM ***REASONING*** The equations of kinematics for rotational motion cannot be used directly to find the angular displacement, because the final angular velocity (not the initial angular velocity), the acceleration, and the time are known. We will combine two of the equations, Equations 8.4 and 8.6 to obtain an expression for the angular displacement that contains the three known variables.

33. SSM WWW *REASONING* The angular displacement of the child when he catches the horse is, from Equation 8.2, $\theta_c = \omega_c t$. In the same time, the angular displacement of the horse is, from Equation 8.7 with $\omega_0 = 0$ rad/s, $\theta_h = \frac{1}{2}\alpha t^2$. If the child is to catch the horse $\theta_c = \theta_h + (\pi/2)$.

SOLUTION Using the above conditions yields

$$\tfrac{1}{2}\alpha t^2 - \omega_c t + \tfrac{1}{2}\pi = 0$$

or

$$\tfrac{1}{2}(0.0100\ \text{rad/s}^2)t^2 - (0.250\ \text{rad/s})t + \tfrac{1}{2}(\pi\ \text{rad}) = 0$$

The quadratic formula yields t = 7.37 s and 42.6 s; therefore, the shortest time needed to catch the horse is $\boxed{t = 7.37\ \text{s}}$.

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34. *REASONING* The angular acceleration α gives rise to a tangential acceleration a_T according to $a_T = r\alpha$ (Equation 8.10). Moreover, it is given that $a_T = g$, where g is the magnitude of the acceleration due to gravity.

SOLUTION Let r be the radial distance of the point from the axis of rotation. Then, according to Equation 8.10, we have

$$\underbrace{g}_{a_T} = r\alpha$$

Thus,

$$r = \frac{g}{\alpha} = \frac{9.80\ \text{m/s}^2}{12.0\ \text{rad/s}^2} = \boxed{0.817\ \text{m}}$$

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35. *REASONING AND SOLUTION*

a. $\omega_A = v/r = (0.381\ \text{m/s})/(0.0508\ \text{m}) = \boxed{7.50\ \text{rad/s}}$

b. $\omega_B = v/r = (0.381\ \text{m/s})(0.114\ \text{m}) = 3.34\ \text{rad/s}$

$$\overline{\alpha} = \frac{\omega_B - \omega_A}{t} = \frac{3.34\ \text{rad/s} - 7.50\ \text{rad/s}}{2.40\times 10^3\ \text{s}} = \boxed{-1.73\times 10^{-3}\ \text{rad/s}^2}$$

The angular velocity is $\boxed{\text{decreasing}}$.

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36. *REASONING* The tangential speed v_T of a point on a rigid body rotating at an angular speed ω is given by $v_T = r\omega$ (Equation 8.9), where r is the radius of the circle described by the moving point. (In this equation ω must be expressed in rad/s.) Therefore, the angular speed of the bacterial motor sought in part *a* is $\omega = v_T/r$. Since we are considering a point on the rim, r is the radius of the motor itself. In part *b*, we seek the elapsed time t for an angular displacement of one revolution at the constant angular velocity ω found in part *a*. We will use $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ (Equation 8.7) to calculate the elapsed time.

SOLUTION
a. The angular speed of the bacterial motor is, from $\omega = v_T/r$ (Equation 8.9),

$$\omega = \frac{v_T}{r} = \frac{2.3\times10^{-5}\ \text{m/s}}{1.5\times10^{-8}\ \text{m}} = \boxed{1500\ \text{rad/s}}$$

b. The bacterial motor is spinning at a constant angular velocity, so it has no angular acceleration. Substituting α = 0 rad/s^2 into $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ (Equation 8.7), and solving for the elapsed time yields

$$\theta = \omega_0 t + \tfrac{1}{2}\left(0\ \text{rad/s}^2\right)t^2 = \omega_0 t \qquad \text{or} \qquad t = \frac{\theta}{\omega_0}$$

The fact that the motor has a constant angular velocity means that its initial and final angular velocities are equal: $\omega_0 = \omega = 1500$ rad/s, the value calculated in part *a*, assuming a counterclockwise or positive rotation. The angular displacement θ is one revolution, or 2π radians, so the elapsed time is

$$t = \frac{\theta}{\omega_0} = \frac{2\pi\ \text{rad}}{1500\ \text{rad/s}} = \boxed{4.2\times10^{-3}\ \text{s}}$$

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37. SSM *REASONING* The angular speed ω and tangential speed v_T are related by Equation 8.9 ($v_T = r\omega$), and this equation can be used to determine the radius r. However, we must remember that this relationship is only valid if we use radian measure. Therefore, it will be necessary to convert the given angular speed in rev/s into rad/s.