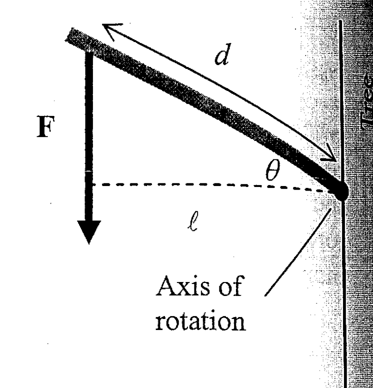
SOLUTION Solving Equation 9.1 for F, we have

$$F = \frac{\text{Magnitude of torque}}{\ell} = \frac{45\ \text{N}\cdot\text{m}}{(0.28\ \text{m})\sin 50.0^\circ} = \boxed{2.1\times10^2\ \text{N}}$$

4. ***REASONING*** The net torque on the branch is the sum of the torques exerted by the children. Each individual torque τ is given by $\tau = F\ell$ (Equation 9.1), where F is the magnitude of the force exerted on the branch by a child, and ℓ is the lever arm (see the diagram). The branch supports each child's weight, so, by Newton's third law, the magnitude F of the force exerted on the branch by each child has the same magnitude as the child's weight: $F = mg$. Both forces are directed downwards. The lever arm for each force is the perpendicular distance between the axis and the force's line of action, so we have $\ell = d\cos\theta$ (see the diagram).



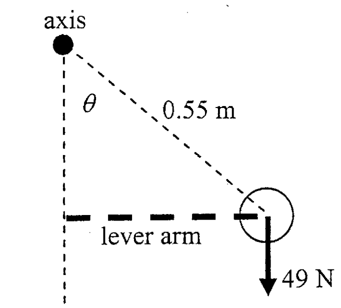
SOLUTION The mass of the first child is $m_1 = 44.0$ kg. This child is a distance $d_1 = 1.30$ m from the tree trunk. The mass of the second child, hanging $d_2 = 2.10$ m from the tree trunk, is $m_2 = 35.0$ kg. Both children produce positive (counterclockwise) torques. The net torque exerted on the branch by the two children is then

$$\Sigma\tau = \tau_1 + \tau_2 = F_1\ell_1 + F_2\ell_2 = \underbrace{m_1 g}_{F_1}\underbrace{d_1\cos\theta}_{\ell_1} + \underbrace{m_2 g}_{F_2}\underbrace{d_2\cos\theta}_{\ell_2} = g\cos\theta\left(m_1 d_1 + m_2 d_1\right)$$

Substituting the given values, we obtain

$$\Sigma\tau = \left(9.80\ \text{m/s}^2\right)\left(\cos 27.0^\circ\right)\left[(44.0\ \text{kg})(1.30\ \text{m}) + (35.0\ \text{kg})(2.10\ \text{m})\right] = \boxed{1140\ \text{N}\cdot\text{m}}$$

5. SSM ***REASONING*** In both parts of the problem, the magnitude of the torque is given by Equation 9.1 as the magnitude F of the force times the lever arm ℓ. In part (a), the lever arm is just the distance of 0.55 m given in the drawing. However, in part (b), the lever arm is less than the given distance and must be expressed using trigonometry as $\ell = (0.55\ \text{m})\sin\theta$. See the drawing at the right.



SOLUTION

a. Using Equation 9.1, we find that

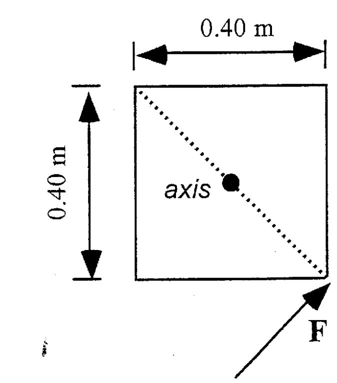
$$\text{Magnitude of torque} = F\ell = (49\ \text{N})(0.55\ \text{m}) = \boxed{27\ \text{N}\cdot\text{m}}$$

b. Again using Equation 9.1, this time with a lever arm of $\ell = (0.55\ \text{m})\sin\theta$, we obtain

$$\text{Magnitude of torque} = 15\ \text{N}\cdot\text{m} = F\ell = (49\ \text{N})(0.55\ \text{m})\sin\theta$$

$$\sin\theta = \frac{15\ \text{N}\cdot\text{m}}{(49\ \text{N})(0.55\ \text{m})} \quad \text{or} \quad \theta = \sin^{-1}\left[\frac{15\ \text{N}\cdot\text{m}}{(49\ \text{N})(0.55\ \text{m})}\right] = \boxed{34^\circ}$$

6. ***REASONING*** The maximum torque will occur when the force is applied perpendicular to the diagonal of the square as shown. The lever arm ℓ is half the length of the diagonal. From the Pythagorean theorem, the lever arm is, therefore,



$$\ell = \tfrac{1}{2}\sqrt{(0.40\ \text{m})^2 + (0.40\ \text{m})^2} = 0.28\ \text{m}$$

Since the lever arm is now known, we can use Equation 9.1 to obtain the desired result directly.

SOLUTION Equation 9.1 gives

$$\tau = F\ell = (15\ \text{N})(0.28\ \text{m}) = \boxed{4.2\ \text{N}\cdot\text{m}}$$

b. Each force produces a counterclockwise rotation. The magnitude of each force is F and each force has a lever arm of $L/2$. Taking counterclockwise as the positive direction, we have

$$\tau = \tau_A + \tau_C = F\left(\frac{L}{2}\right) + F\left(\frac{L}{2}\right) = \boxed{FL}$$

c. When the axis passes through point C, the torque due to the force at C is zero. The lever arm for the force at A is L. Therefore, taking counterclockwise as the positive direction, we have

$$\tau = \tau_A + \tau_C = FL + 0 = \boxed{FL}$$

Note that the value of the torque produced by the couple is the same in all three cases; in other words, when the couple acts on the tire wrench, the couple produces a torque that does *not* depend on the location of the axis.

---

10. ***REASONING AND SOLUTION*** The net torque about the axis in text drawing (a) is

$$\Sigma\tau = \tau_1 + \tau_2 = F_1 b - F_2 a = 0$$

Considering that $F_2 = 3F_1$, we have $b - 3a = 0$. The net torque in drawing (b) is then

$$\Sigma\tau = F_1(1.00\text{ m} - a) - F_2 b = 0 \quad \text{or} \quad 1.00\text{ m} - a - 3b = 0$$

Solving the first equation for b, substituting into the second equation and rearranging, gives

$$a = \boxed{0.100\text{ m}} \quad \text{and} \quad b = \boxed{0.300\text{ m}}$$

---

11. ***REASONING*** Although this arrangement of body parts is vertical, we can apply Equation 9.3 to locate the overall center of gravity by simply replacing the horizontal position x by the vertical position y, as measured relative to the floor.

SOLUTION Using Equation 9.3, we have

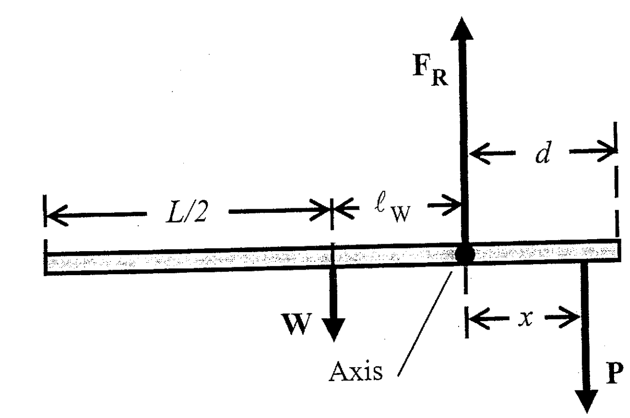
$$y_{cg} = \frac{W_1 y_1 + W_2 y_2 + W_3 y_3}{W_1 + W_2 + W_3}$$

$$= \frac{(438\text{ N})(1.28\text{ m}) + (144\text{ N})(0.760\text{ m}) + (87\text{ N})(0.250\text{ m})}{438\text{ N} + 144\text{ N} + 87\text{ N}} = \boxed{1.03\text{ m}}$$

---

12. ***REASONING*** At every instant before the plank begins to tip, it is in equilibrium, and the net torque on it is zero: $\Sigma\tau = 0$ (Equation 9.2). When the person reaches the maximum distance x along the overhanging part, the plank is just about to rotate about the right support. At that instant, the plank loses contact with the left support, which consequently exerts no force on it. This leaves only three vertical forces acting on the plank: the weight **W** of the plank, the force $\mathbf{F_R}$ due to the right support, and the force **P** due to the person (see the free-body diagram of the plank). The force $\mathbf{F_R}$ acts at the right support, which we take as the axis, so its lever arm is zero. The lever arm for the force **P** is the distance x. Since counterclockwise is the positive direction, Equation 9.2 gives

$$\Sigma\tau = W\ell_W - Px = 0 \quad \text{or} \quad x = \frac{W\ell_W}{P} \tag{1}$$



Free-body diagram of the plank

SOLUTION The weight W = 225 N of the plank is known, and the force P due to the person is equal to the person's weight: P = 450 N. This is because the plank supports the person against the pull of gravity, and Newton's third law tells us that the person and the plank exert forces of equal magnitude on each other. The plank's weight W acts at the center of the uniform plank, so we have (see the drawing)

$$\ell_W + d = \tfrac{1}{2}L \quad \text{or} \quad \ell_W = \tfrac{1}{2}L - d \tag{2}$$

where $d = 1.1$ m is the length of the overhanging part of the plank, and $L = 5.0$ m is the length of the entire plank.

Substituting Equation (2) into Equation (1), we obtain

$$x = \frac{W\left(\frac{1}{2}L - d\right)}{P} = \frac{(225\text{ N})\left[\frac{1}{2}(5.0\text{ m}) - 1.1\text{ m}\right]}{450\text{ N}} = \boxed{0.70\text{ m}}$$

Left design $\Sigma\tau = -(525\text{ N})(0.400\text{ m}) - (60.0\text{ N})(0.600\text{ m}) + F(1.300\text{ m}) = 0$

$$F = \frac{(525\text{ N})(0.400\text{ m}) + (60.0\text{ N})(0.600\text{ m})}{1.300\text{ m}} = \boxed{189\text{ N}}$$

Right design $\Sigma\tau = -(60.0\text{ N})(0.600\text{ m}) + F(1.300\text{ m}) = 0$

$$F = \frac{(60.0\text{ N})(0.600\text{ m})}{1.300\text{ m}} = \boxed{27.7\text{ N}}$$

---

19. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The net torque about an axis through the contact point between the tray and the thumb is

$$\Sigma\tau = F(0.0400\text{ m}) - (0.250\text{ kg})(9.80\text{ m/s}^2)(0.320\text{ m}) - (1.00\text{ kg})(9.80\text{ m/s}^2)(0.180\text{ m}) - (0.200\text{ kg})(9.80\text{ m/s}^2)(0.140\text{ m}) = 0$$

$$F = \boxed{70.6\text{ N, up}}$$

Similarly, the net torque about an axis through the point of contact between the tray and the finger is

$$\Sigma\tau = T(0.0400\text{ m}) - (0.250\text{ kg})(9.80\text{ m/s}^2)(0.280\text{ m}) - (1.00\text{ kg})(9.80\text{ m/s}^2)(0.140\text{ m}) - (0.200\text{ kg})(9.80\text{ m/s}^2)(0.100\text{ m}) = 0$$

$$T = \boxed{56.4\text{ N, down}}$$

---

20. ***REASONING*** The jet is in equilibrium, so the sum of the external forces is zero, and the sum of the external torques is zero. We can use these two conditions to evaluate the forces exerted on the wheels.

SOLUTION
a. Let F_f be the magnitude of the normal force that the ground exerts on the front wheel. Since the net torque acting on the plane is zero, we have (using an axis through the points of contact between the rear wheels and the ground)

$$\Sigma\tau = -W\ell_w + F_f\ell_f = 0$$

where W is the weight of the plane, and ℓ_w and ℓ_f are the lever arms for the forces W and F_f, respectively. Thus,

$$\Sigma\tau = -(1.00\times10^6\text{ N})(15.0\text{ m} - 12.6\text{ m}) + F_f(15.0\text{ m}) = 0$$

Solving for F_f gives $F_f = \boxed{1.60\times10^5\text{ N}}$.

b. Setting the sum of the vertical forces equal to zero yields

$$\Sigma F_y = F_f + 2F_r - W = 0$$

where the factor of 2 arises because there are two rear wheels. Substituting the data gives

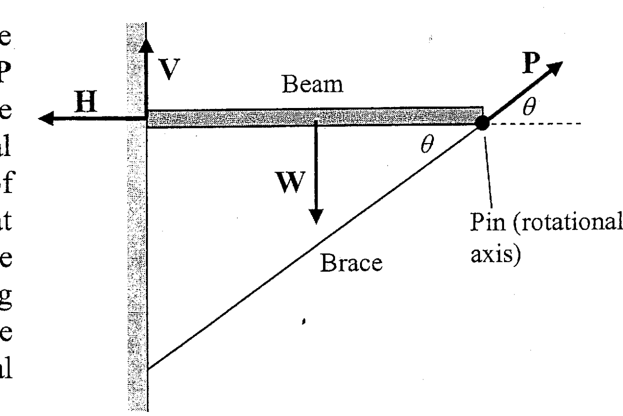
$$\Sigma F_y = 1.60\times10^5\text{ N} + 2F_r - 1.00\times10^6\text{ N} = 0$$

$$F_r = \boxed{4.20\times10^5\text{ N}}$$

---

21. ***REASONING*** Since the beam is in equilibrium, the sum of the horizontal and vertical forces must be zero: $\Sigma F_x = 0$ and $\Sigma F_y = 0$ (Equations 9.4a and b). In addition, the net torque about any axis of rotation must also be zero: $\Sigma\tau = 0$ (Equation 9.2).

SOLUTION The drawing shows the beam, as well as its weight **W**, the force **P** that the pin exerts on the right end of the beam, and the horizontal and vertical forces, **H** and **V**, applied to the left end of the beam by the hinge. Assuming that upward and to the right are the positive directions, we obtain the following expressions by setting the sum of the vertical and the sum of the horizontal forces equal to zero:



Horizontal forces $P\cos\theta - H = 0$ (1)

Vertical forces $P\sin\theta + V - W = 0$ (2)

Using a rotational axis perpendicular to the plane of the paper and passing through the pin, and remembering that counterclockwise torques are positive, we also set the sum of the torques equal to zero. In doing so, we use L to denote the length of the beam and note that the lever arms for **W** and **V** are $\frac{1}{2}L$ and L, respectively. The forces **P** and **H** create no torques relative to this axis, because their lines of action pass directly through it.

Torques $$W\left(\tfrac{1}{2}L\right)-VL=0 \qquad (3)$$

Since L can be eliminated algebraically, Equation (3) may be solved immediately for V:

$$V=\tfrac{1}{2}W=\tfrac{1}{2}(340\text{ N})=\boxed{170\text{ N}}$$

Substituting this result into Equation (2) gives

$$P\sin\theta+\tfrac{1}{2}W-W=0$$

$$P=\frac{W}{2\sin\theta}=\frac{340\text{ N}}{2\sin 39^\circ}=\boxed{270\text{ N}}$$

Substituting this result into Equation (1) yields

$$\left(\frac{W}{2\sin\theta}\right)\cos\theta-H=0$$

$$H=\frac{W}{2\tan\theta}=\frac{340\text{ N}}{2\tan 39^\circ}=\boxed{210\text{ N}}$$

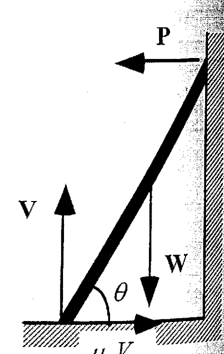
---

22. ***REASONING AND SOLUTION*** The net torque about an axis through the elbow joint is

$$\Sigma\tau=(111\text{ N})(0.300\text{ m})-(22.0\text{ N})(0.150\text{ m})-(0.0250\text{ m})M=0 \quad\text{or}\quad M=\boxed{1.20\times10^3\text{ N}}$$

---

23. **SSM** ***REASONING*** The drawing shows the forces acting on the board, which has a length L. The ground exerts the vertical normal force **V** on the lower end of the board. The maximum force of static friction has a magnitude of $\mu_s V$ and acts horizontally on the lower end of the board. The weight **W** acts downward at the board's center. The vertical wall applies a force **P** to the upper end of the board, this force being perpendicular to the wall since the wall is smooth (i.e., there is no friction along the wall). We take upward and to the right as our positive directions. Then, since the horizontal forces balance to zero, we have



$$\mu_s V-P=0 \qquad (1)$$

The vertical forces also balance to zero giving

$$V-W=0 \qquad (2)$$

Using an axis through the lower end of the board, we express the fact that the torques balance to zero as

$$PL\sin\theta-W\left(\frac{L}{2}\right)\cos\theta=0 \qquad (3)$$

Equations (1), (2), and (3) may then be combined to yield an expression for θ.

SOLUTION Rearranging Equation (3) gives

$$\tan\theta=\frac{W}{2P} \qquad (4)$$

But, $P=\mu_s V$ according to Equation (1), and $W=V$ according to Equation (2). Substituting these results into Equation (4) gives

$$\tan\theta=\frac{V}{2\mu_s V}=\frac{1}{2\mu_s}$$

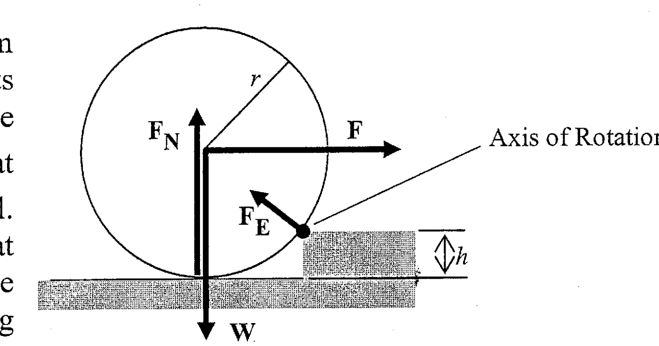
Therefore,

$$\theta=\tan^{-1}\left(\frac{1}{2\mu_s}\right)=\tan^{-1}\left[\frac{1}{2(0.650)}\right]=\boxed{37.6^\circ}$$

---

24. ***REASONING*** When the wheel is resting on the ground it is in equilibrium, so the sum of the torques about any axis of rotation is zero $(\Sigma\tau=0)$. This equilibrium condition will provide us with a relation between the magnitude of **F** and the normal force that the ground exerts on the wheel. When **F** is large enough, the wheel will rise up off the ground, and the normal force will become zero. From our relation, we can determine the magnitude of **F** when this happens.

SOLUTION The free body diagram shows the forces acting on the wheel: its weight **W**, the normal force $\mathbf{F}_N$, the horizontal force **F**, and the force $\mathbf{F}_E$ that the edge of the step exerts on the wheel. We select the axis of rotation to be at the edge of the step, so that the torque produced by $\mathbf{F}_E$ is zero. Letting ℓ_N, ℓ_W, and ℓ_F represent the lever arms for the forces $\mathbf{F}_N$, **W**, and **F**, the sum of the torques is



$$\Sigma\tau=-F_N\underbrace{\sqrt{r^2-(r-h)^2}}_{\ell_N}+W\underbrace{\sqrt{r^2-(r-h)^2}}_{\ell_W}-F\underbrace{(r-h)}_{\ell_F}=0$$

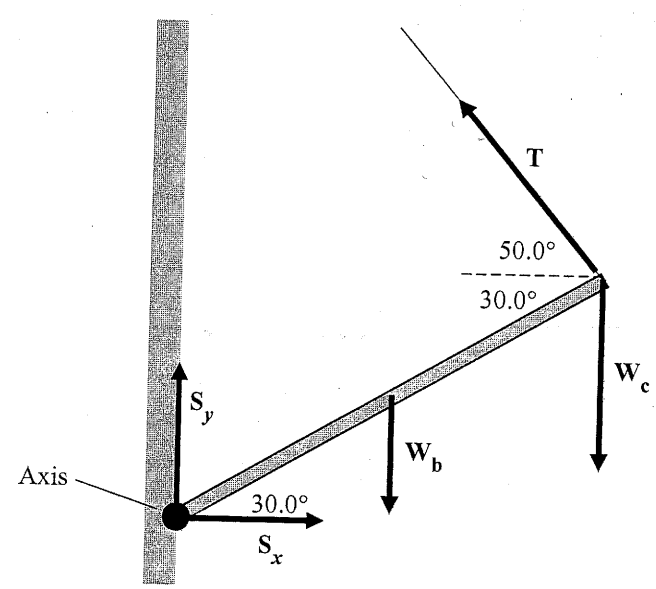
Solving this equation for F gives

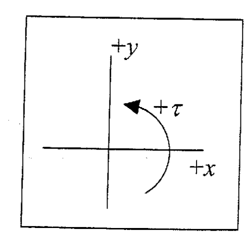
$$F = \frac{(W - F_N)\sqrt{r^2 - (r-h)^2}}{r-h}$$

When the bicycle wheel just begins to lift off the ground the normal force becomes zero (F_N = 0 N). When this happens, the magnitude of **F** is

$$F = \frac{(W - F_N)\sqrt{r^2 - (r-h)^2}}{r-h} = \frac{(25.0\text{ N} - 0\text{ N})\sqrt{(0.340\text{ m})^2 - (0.340\text{ m} - 0.120\text{ m})^2}}{0.340\text{ m} - 0.120\text{ m}} = \boxed{29\text{ N}}$$

25. ***REASONING*** The drawing shows the beam and the five forces that act on it: the horizontal and vertical components $\mathbf{S}_x$ and $\mathbf{S}_y$ that the wall exerts on the left end of the beam, the weight $\mathbf{W}_b$ of the beam, the force due to the weight $\mathbf{W}_c$ of the crate, and the tension **T** in the cable. The beam is uniform, so its center of gravity is at the center of the beam, which is where its weight can be assumed to act. Since the beam is in equilibrium, the sum of the torques about any axis of rotation must be zero $(\Sigma\tau = 0)$, and the sum of the forces in the horizontal and vertical directions must be zero $(\Sigma F_x = 0,\ \Sigma F_y = 0)$. These three conditions will allow us to determine the magnitudes of $\mathbf{S}_x$, $\mathbf{S}_y$, and **T**.





SOLUTION
a. We will begin by taking the axis of rotation to be at the left end of the beam. Then the torques produced by $\mathbf{S}_x$ and $\mathbf{S}_y$ are zero, since their lever arms are zero. When we set the sum of the torques equal to zero, the resulting equation will have only one unknown, T, in it. Setting the sum of the torques produced by the three forces equal to zero gives (with L equal to the length of the beam)

$$\Sigma\tau = -W_b\left(\tfrac{1}{2}L\cos 30.0°\right) - W_c\left(L\cos 30.0°\right) + T\left(L\sin 80.0°\right) = 0$$

Algebraically eliminating L from this equation and solving for T gives

$$T = \frac{W_b\left(\tfrac{1}{2}\cos 30.0°\right) + W_c\left(\cos 30.0°\right)}{\sin 80.0°}$$

$$= \frac{(1220\text{ N})\left(\tfrac{1}{2}\cos 30.0°\right) + (1960\text{ N})(\cos 30.0°)}{\sin 80.0°} = \boxed{2260\text{ N}}$$

b. Since the beam is in equilibrium, the sum of the forces in the vertical direction is zero:

$$\Sigma F_y = +S_y - W_b - W_c + T\sin 50.0° = 0$$

Solving for S_y gives

$$S_y = W_b + W_c - T\sin 50.0° = 1220\text{ N} + 1960\text{ N} - (2260\text{ N})\sin 50.0° = \boxed{1450\text{ N}}$$

The sum of the forces in the horizontal direction must also be zero:

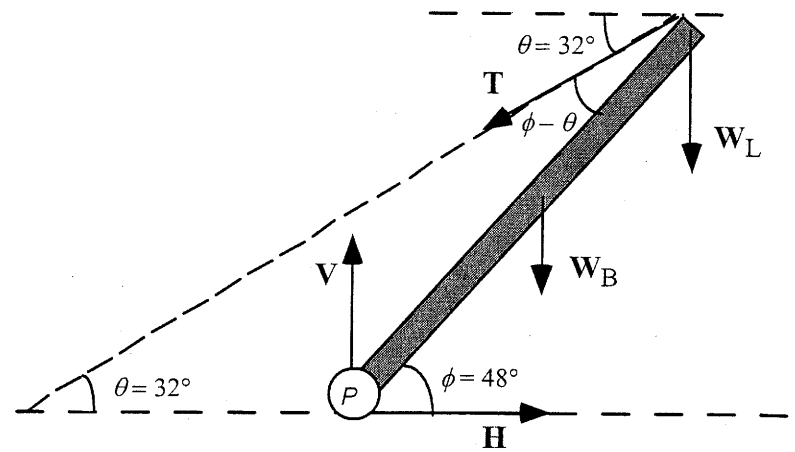
$$\Sigma F_x = +S_x - T\cos 50.0° = 0$$

so that

$$S_x = T\cos 50.0° = (2260\text{ N})\cos 50.0° = \boxed{1450\text{ N}}$$

26. ***REASONING*** If we assume that the system is in equilibrium, we know that the vector sum of all the forces, as well as the vector sum of all the torques, that act on the system must be zero.

The figure below shows a free body diagram for the boom. Since the boom is assumed to be uniform, its weight $\mathbf{W}_B$ is located at its center of gravity, which coincides with its geometrical center. There is a tension **T** in the cable that acts at an angle θ to the horizontal, as shown. At the hinge pin P, there are two forces acting. The vertical force **V** that acts on the end of the boom prevents the boom from falling down. The horizontal force **H** that also acts at the hinge pin prevents the boom from sliding to the left. The weight $\mathbf{W}_L$ of the wrecking ball (the "load") acts at the end of the boom.



By applying the equilibrium conditions to the boom, we can determine the desired forces.

SOLUTION The directions upward and to the right will be taken as the positive directions. In the x direction we have

$$\sum F_x = H - T\cos\theta = 0 \qquad (1)$$

while in the y direction we have

$$\sum F_y = V - T\sin\theta - W_\text{L} - W_\text{B} = 0 \qquad (2)$$

Equations (1) and (2) give us two equations in three unknown. We must, therefore, find a third equation that can be used to determine one of the unknowns. We can get the third equation from the torque equation.

In order to write the torque equation, we must first pick an axis of rotation and determine the lever arms for the forces involved. Since both V and H are unknown, we can eliminate them from the torque equation by picking the rotation axis through the point P (then both V and H have zero lever arms). If we let the boom have a length L, then the lever arm for $\mathbf{W}_\text{L}$ is $L\cos\phi$, while the lever arm for $\mathbf{W}_\text{B}$ is $(L/2)\cos\phi$. From the figure, we see that the lever arm for $\mathbf{T}$ is $L\sin(\phi-\theta)$. If we take counterclockwise torques as positive, then the torque equation is

$$\Sigma\tau = -W_\text{B}\left(\frac{L\cos\phi}{2}\right) - W_\text{L}L\cos\phi + TL\sin(\phi-\theta) = 0$$

Solving for T, we have

$$T = \frac{\frac{1}{2}W_\text{B} + W_\text{L}}{\sin(\phi-\theta)}\cos\phi \qquad (3)$$

a. From Equation (3) the tension in the support cable is

$$T = \frac{\frac{1}{2}(3600\ \text{N}) + 4800\ \text{N}}{\sin(48° - 32°)}\cos 48° = \boxed{1.6\times10^4\ \text{N}}$$

b. The force exerted on the lower end of the hinge at the point P is the vector sum of the forces $\mathbf{H}$ and $\mathbf{V}$. According to Equation (1),

$$H = T\cos\theta = \left(1.6\times10^4\ \text{N}\right)\cos 32° = 1.4\times10^4\ \text{N}$$

and, from Equation (2)

$$V = W_L + W_B + T\sin\theta = 4800\ \text{N} + 3600\ \text{N} + \left(1.6\times10^4\ \text{N}\right)\sin 32° = 1.7\times10^4\ \text{N}$$

Since the forces $\mathbf{H}$ and $\mathbf{V}$ are at right angles to each other, the magnitude of their vector sum can be found from the Pythagorean theorem:

$$F_\text{P} = \sqrt{H^2+V^2} = \sqrt{(1.4\times10^4\ \text{N})^2 + (1.7\times10^4\ \text{N})^2} = \boxed{2.2\times10^4\ \text{N}}$$

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27. SSM ***REASONING*** Since the man holds the ball motionless, the ball and the arm are in equilibrium. Therefore, the net force, as well as the net torque about any axis, must be zero.

SOLUTION Using Equation 9.1, the net torque about an axis through the elbow joint is

$$\Sigma\tau = M(0.0510\ \text{m}) - (22.0\ \text{N})(0.140\ \text{m}) - (178\ \text{N})(0.330\ \text{m}) = 0$$

Solving this expression for M gives $M = \boxed{1.21\times10^3\ \text{N}}$.

The net torque about an axis through the center of gravity is

$$\Sigma\tau = -(1210\ \text{N})(0.0890\ \text{m}) + F(0.140\ \text{m}) - (178\ \text{N})(0.190\ \text{m}) = 0$$

Solving this expression for F gives $F = \boxed{1.01\times10^3\ \text{N}}$. Since the forces must add to give a net force of zero, we know that the direction of $\mathbf{F}$ is $\boxed{\text{downward}}$.

---

28. ***REASONING*** Although the crate is in translational motion, it undergoes no angular acceleration. Therefore, the net torque acting on the crate must be zero: $\Sigma\tau = 0$ (Equation 9.2). The four forces acting on the crate appear in the free-body diagram: its weight $\mathbf{W}$, the kinetic friction force $\mathbf{f}_k$, the normal force $\mathbf{F}_N$, and the tension $\mathbf{T}$ in the strap. We will take the edge of the crate sliding along the floor as the rotation axis for applying Equation 9.2. Both the friction force and the normal force act at this point. These two forces, therefore, generate no torque about the axis of rotation, so the clockwise torque of the crate's weight $\mathbf{W}$ must balance the counterclockwise torque of the tension $\mathbf{T}$ in the strap: