$$\frac{v_{\text{longer}}}{v_{\text{shorter}}} = \frac{\sqrt{\dfrac{F}{m/L_{\text{longer}}}}}{\sqrt{\dfrac{F}{m/L_{\text{shorter}}}}} = \sqrt{\frac{L_{\text{longer}}}{L_{\text{shorter}}}}$$

Noting that $v_{\text{shorter}} = 240$ m/s and that $L_{\text{longer}} = 2L_{\text{shorter}}$, the speed of the wave on the longer wire is

$$v_{\text{longer}} = v_{\text{shorter}}\sqrt{\frac{L_{\text{longer}}}{L_{\text{shorter}}}} = (240\text{ m/s})\sqrt{\frac{2L_{\text{shorter}}}{L_{\text{shorter}}}} = (240\text{ m/s})\sqrt{2} = \boxed{340\text{ m/s}}$$

13. SSM ***REASONING*** The tension F in the violin string can be found by solving Equation 16.2 for F to obtain $F = mv^2/L$, where v is the speed of waves on the string and can be found from Equation 16.1 as $v = f\lambda$.

SOLUTION Combining Equations 16.2 and 16.1 and using the given data, we obtain

$$F = \frac{mv^2}{L} = (m/L)f^2\lambda^2 = \left(7.8\times10^{-4}\text{ kg/m}\right)(440\text{ Hz})^2\left(65\times10^{-2}\text{ m}\right)^2 = \boxed{64\text{ N}}$$

14. ***REASONING*** The length L of the string is one of the factors that affects the speed of a wave traveling on it, in so far as the speed v depends on the mass per unit length m/L according to $v = \sqrt{\dfrac{F}{m/L}}$ (Equation 16.2). The other factor affecting the speed is the tension F. The speed is not directly given here. However, the frequency f and the wavelength λ are given, and the speed is related to them according to $v = f\lambda$ (Equation 16.1). Substituting Equation 16.1 into Equation 16.2 will give us an equation that can be solved for the length L.

SOLUTION Substituting Equation 16.1 into Equation 16.2 gives

$$v = f\lambda = \sqrt{\frac{F}{m/L}}$$

Solving for the length L, we find that

$$L = \frac{f^2\lambda^2 m}{F} = \frac{(260\text{ Hz})^2(0.60\text{ m})^2\left(5.0\times10^{-3}\text{ kg}\right)}{180\text{ N}} = \boxed{0.68\text{ m}}$$

REASONING The speed v of the transverse pulse on the wire is determined by the tension F in the wire and the mass per unit length m/L of the wire, according to $v = \sqrt{\dfrac{F}{m/L}}$ (Equation 16.2). The ball has a mass M. Since the wire supports the weight Mg of the ball and since the weight of the wire is negligible, it is only the ball's weight that determines the tension in the wire, $F = Mg$. Therefore, we can use Equation 16.2 with this value of the tension and solve it for the acceleration g due to gravity. The speed of the transverse pulse is not given, but we know that the pulse travels the length L of the wire in a time t and that the speed is $v = L/t$.

SOLUTION Substituting the tension $F = Mg$ and the speed $v = L/t$ into Equation 16.2 for the speed of the pulse on the string gives

$$v = \sqrt{\frac{F}{m/L}} \quad \text{or} \quad \frac{L}{t} = \sqrt{\frac{Mg}{m/L}}$$

Solving for the acceleration g due to gravity, we obtain

$$g = \frac{\left(\dfrac{L}{t}\right)^2(m/L)}{M} = \frac{\left(\dfrac{0.95\text{ m}}{0.016\text{ s}}\right)^2\left(1.2\times10^{-4}\text{ kg/m}\right)}{0.055\text{ kg}} = \boxed{7.7\text{ m/s}^2}$$

16. ***REASONING*** Each pulse travels a distance that is given by vt, where v is the wave speed and t is the travel time up to the point when they pass each other. The sum of the distances traveled by each pulse must equal the 50.0-m length of the wire, since each pulse starts out from opposite ends of the wires.

SOLUTION Using v_A and v_B to denote the speeds on either wire, we have

$$v_A t + v_B t = 50.0\text{ m}$$

Solving for the time t and using Equation 16.2 $\left(v = \sqrt{\dfrac{F}{m/L}}\right)$, we find

$$t = \frac{50.0\text{ m}}{v_A + v_B} = \frac{50.0\text{ m}}{\sqrt{\dfrac{F_A}{m/L}} + \sqrt{\dfrac{F_B}{m/L}}} = \frac{50.0\text{ m}}{\sqrt{\dfrac{6.00\times10^2\text{ N}}{0.020\text{ kg/m}}} + \sqrt{\dfrac{3.00\times10^2\text{ N}}{0.020\text{ kg/m}}}} = \boxed{0.17\text{ s}}$$

$$v = \sqrt{\frac{m\left(\frac{y}{L}\right)g}{m/L}} = \sqrt{yg}$$

b. Using the expression just derived, we find the following speeds

[y = 0.50 m] $v = \sqrt{yg} = \sqrt{(0.50\text{ m})(9.80\text{ m/s}^2)} = \boxed{2.2\text{ m/s}}$

[y = 2.0 m] $v = \sqrt{yg} = \sqrt{(2.0\text{ m})(9.80\text{ m/s}^2)} = \boxed{4.4\text{ m/s}}$

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22. ***REASONING AND SOLUTION*** According to Equation 16.2, the tension in the wire initially is

$$F_0 = (m/L)v^2 = (9.8 \times 10^{-3}\text{ kg/m})(46\text{ m/s})^2 = 21\text{ N}$$

As the temperature is lowered, the wire will attempt to shrink by an amount $\Delta L = \alpha L \Delta T$ (Equation 12.2), where α is the coefficient of thermal expansion. Since the wire cannot shrink, a stress will develop (see Equation 10.17 and Section 10.8), according to

$$\text{Stress} = Y\Delta L/L = Y\alpha\Delta T$$

where Y is Young's modulus. This stress corresponds (see Section 10.8) to an additional tension F':

$$F' = (\text{Stress})A = YA\alpha\Delta T = (1.1 \times 10^{11}\text{ Pa})(1.1 \times 10^{-6}\text{ m}^2)(17 \times 10^{-6}/\text{C}°)(14\text{ C}°) = 29\text{ N}$$

The total tension in the wire at the lower temperature is now $F = F_0 + F'$, so that the new speed of the waves on the wire is

$$v = \sqrt{\frac{F_0 + F'}{m/L}} = \sqrt{\frac{21\text{ N} + 29\text{ N}}{9.8 \times 10^{-3}\text{ kg/m}}} = \boxed{71\text{ m/s}}$$

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23. ***REASONING AND SOLUTION*** If the string has length L, the time required for a wave on the string to travel from the center of the circle to the ball is

$$t = \frac{L}{v_{\text{wave}}} \qquad (1)$$

The speed of the wave is given by text Equation 16.2

$$v_{\text{wave}} = \sqrt{\frac{F}{m_{\text{string}}/L}} \qquad (2)$$

The tension F in the string provides the centripetal force on the ball, so that

$$F = m_{\text{ball}}\omega^2 r = m_{\text{ball}}\omega^2 L \qquad (3)$$

Eliminating the tension F from Equations (2) and (3) above yields

$$v_{\text{wave}} = \sqrt{\frac{m_{\text{ball}}\omega^2 L}{m_{\text{string}}/L}} = \sqrt{\frac{m_{\text{ball}}\omega^2 L^2}{m_{\text{string}}}} = L\sqrt{\frac{m_{\text{ball}}\omega^2}{m_{\text{string}}}}$$

Substituting this expression for v_{wave} into Equation (1) gives

$$t = \frac{L}{L\sqrt{\frac{m_{\text{ball}}\omega^2}{m_{\text{string}}}}} = \sqrt{\frac{m_{\text{string}}}{m_{\text{ball}}\omega^2}} = \sqrt{\frac{0.0230\text{ kg}}{(15.0\text{ kg})(12.0\text{ rad/s})^2}} = \boxed{3.26 \times 10^{-3}\text{ s}}$$

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24. ***REASONING*** The speed v of the wave is related to its wavelength λ and frequency f according to $v = f\lambda$ (Equation 16.1), so we will need to determine the wavelength and frequency. The mathematical description of this wave has the form $y = A\sin\left(2\pi ft + \frac{2\pi x}{\lambda}\right)$ (Equation 16.4), which applies to waves moving in the - x direction. Identifying like terms in the given mathematical description of the wave and Equation 16.4 will permit us to determine the wavelength λ and frequency f.

SOLUTION The dimensionless term $2\pi ft$ in Equation 16.4 corresponds to the term $8.2\pi t$ in the given wave equation. The time t is measured in seconds (s), so in order for the quantity $8.2\pi t$ to be dimensionless, the units of the numerical factor 8.2 must be s^{-1} = Hz, and we have that

$$2\cancel{\pi} f \cancel{t} = (8.2\text{ Hz})\cancel{\pi}\cancel{t} \quad \text{or} \quad 2f = 8.2\text{ Hz} \quad \text{or} \quad f = \frac{8.2\text{ Hz}}{2} = 4.1\text{ Hz}$$

Similarly, the dimensionless term $2\pi x/\lambda$ in Equation 16.4 corresponds to the term $0.54\pi x$ in the mathematical description of this wave. Because x is measured in meters (m), the term $0.54\pi x$ is dimensionless if the numerical factor 0.54 has units of m^{-1}. Thus,

$$\frac{2\cancel{\pi}\cancel{x}}{\lambda}=\left(0.54\text{ m}^{-1}\right)\cancel{\pi}\cancel{x} \quad \text{or} \quad \frac{2}{\lambda}=0.54\text{ m}^{-1} \quad \text{or} \quad \lambda=\frac{2}{0.54\text{ m}^{-1}}=3.7\text{ m}$$

Applying $v=f\lambda$ (Equation 16.1), we obtain the speed of the wave:

$$v=\lambda f=(3.7\text{ m})(4.1\text{ Hz})=\boxed{15\text{ m/s}}$$

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25. SSM ***REASONING*** Since the wave is traveling in the $+x$ direction, its form is given by Equation 16.3 as

$$y=A\sin\left(2\pi ft-\frac{2\pi x}{\lambda}\right)$$

We are given that the amplitude is A = 0.35 m. However, we need to evaluate $2pf$ and $\frac{2\pi}{\lambda}$.

Although the wavelength λ is not stated directly, it can be obtained from the values for the speed v and the frequency f, since we know that $v=f\lambda$ (Equation 16.1).

SOLUTION Since the frequency is f = 14 Hz, we have

$$2pf = 2p(14\text{ Hz}) = 88\text{ rad/s}$$

It follows from Equation 16.1 that

$$\frac{2\pi}{\lambda}=\frac{2\pi f}{v}=\frac{2\pi(14\text{ Hz})}{5.2\text{ m/s}}=17\text{ m}^{-1}$$

Using these values for $2pf$ and $\frac{2\pi}{\lambda}$ in Equation 16.3, we have

$$y=A\sin\left(2\pi ft-\frac{2\pi x}{\lambda}\right)$$

$$\boxed{y=(0.35\text{ m})\sin\left[(88\text{ rad/s})t-\left(17\text{ m}^{-1}\right)x\right]}$$

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26. ***REASONING AND SOLUTION*** Referring to the drawing that accompanies the problem statement, we find from the graph on the left that λ = 0.060 m – 0.020 m = 0.040 m and A = 0.010 m; from the graph on the right we find that T = 0.30 s – 0.10 s = 0.20 s. Then, f = 1/(0.20 s) = 5.0 Hz. Substituting these into Equation 16.3 we get

$$y=A\sin\left(2\pi ft-\frac{2\pi x}{\lambda}\right) \quad \text{and} \quad \boxed{y=(0.010\text{ m})\sin(10\pi t-50\pi x)}$$

REASONING The mathematical form for the displacement of a wave traveling in the $-x$ direction is given by Equation 16.4: $y=A\sin\left(2\pi ft+\frac{2\pi x}{\lambda}\right)$.

SOLUTION Using Equation 16.1 and the fact that $f=1/T$, we obtain the following numerical values for f and λ:

$$f=\frac{1}{T}=\frac{1}{0.77\text{ s}}=1.3\text{ Hz} \quad \text{and} \quad \lambda=\frac{v}{f}=\frac{12\text{ m/s}}{1.3\text{ Hz}}=9.2\text{ m}$$

Using the given value for the amplitude A and substituting the above values for f and λ into Equation 16.4 gives

$$y=A\sin\left(2\pi ft+\frac{2\pi x}{\lambda}\right)=(0.37\text{ m})\sin\left[2\pi(1.3\text{ Hz})t+\left(\frac{2\pi}{9.2\text{ m}}\right)x\right]$$

$$\boxed{y=(0.37\text{ m})\sin\left[(8.2\text{ rad/s})t+\left(0.68\text{ m}^{-1}\right)x\right]}$$

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28. ***REASONING*** The tension F in a string can be found from $v=\sqrt{\frac{F}{m/L}}$ (Equation 16.2), where v is the speed of a wave traveling along the string and m/L is the string's linear density. The speed v of the wave depends upon the frequency f and wavelength λ of the wave, according to $v=f\lambda$ (Equation 16.1). We will determine the frequency and wavelength of the wave by comparing the given equation for the displacement y of a particle from its equilibrium position to the general equation $y=A\sin\left(2\pi ft-\frac{2\pi x}{\lambda}\right)$ (Equation 16.3). Once we have identified the frequency and wavelength, we will be able to determine the wave speed v via Equation 16.1. We will then use Equation 16.2 to obtain the tension F in the string.

SOLUTION Comparing $y=(0.021\text{ m})\sin(25t-2.0x)$ with the general equation $y=A\sin\left(2\pi ft-\frac{2\pi x}{\lambda}\right)$ (Equation 16.3), we see that $2\pi ft=25t$, or $2\pi f=25$, and that $\frac{2\pi x}{\lambda}=2.0x$, or $\frac{2\pi}{\lambda}=2.0$. Therefore, we have that

$$f=\frac{25}{2\pi}\text{ Hz} \quad \text{and} \quad \lambda=\frac{2\pi}{2.0}\text{ m}=\pi\text{ m} \qquad (1)$$

Squaring both sides of $v = \sqrt{\dfrac{F}{m/L}}$ (Equation 16.2) and solving for F, we obtain

$$v^2 = \frac{F}{m/L} \qquad \text{or} \qquad F = v^2\left(m/L\right) \tag{2}$$

Substituting $v = f\lambda$ (Equation 16.1) into Equation (2) yields

$$F = v^2\left(m/L\right) = \left(f\lambda\right)^2\left(m/L\right) \tag{3}$$

Substituting Equations (1) and the linear density into Equation (3), we find that

$$F = \left(f\lambda\right)^2\left(m/L\right) = \left[\left(\frac{25}{2\cancel{\pi}}\ \text{Hz}\right)\left(\cancel{\pi}\ \text{m}\right)\right]^2\left(1.6\times10^{-2}\ \text{kg/m}\right) = \boxed{2.5\ \text{N}}$$

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29. $\boxed{\text{SSM}}$ ***REASONING*** The speed of a wave on the string is given by Equation 16.2 as $v = \sqrt{\dfrac{F}{m/L}}$, where F is the tension in the string and m/L is the mass per unit length (or linear density) of the string. The wavelength λ is the speed of the wave divided by its frequency f (Equation 16.1).

SOLUTION
a. The speed of the wave on the string is

$$v = \sqrt{\frac{F}{\left(m/L\right)}} = \sqrt{\frac{15\ \text{N}}{0.85\ \text{kg/m}}} = \boxed{4.2\ \text{m/s}}$$

b. The wavelength is

$$\lambda = \frac{v}{f} = \frac{4.2\ \text{m/s}}{12\ \text{Hz}} = \boxed{0.35\ \text{m}}$$

c. The amplitude of the wave is $A = 3.6\ \text{cm} = 3.6\times10^{-2}$ m. Since the wave is moving along the $-x$ direction, the mathematical expression for the wave is given by Equation 16.4 as

$$y = A\sin\left(2\pi f t + \frac{2\pi x}{\lambda}\right)$$

Substituting in the numbers for A, f, and λ, we have

$$y = A\sin\left(2\pi f t + \frac{2\pi x}{\lambda}\right) = \left(3.6\times10^{-2}\ \text{m}\right)\sin\left[2\pi\left(12\ \text{Hz}\right)t + \frac{2\pi x}{0.35\ \text{m}}\right]$$

$$= \boxed{\left(3.6\times10^{-2}\ \text{m}\right)\sin\left[\left(75\ \text{rad/s}\right)t + \left(18\ \text{m}^{-1}\right)x\right]}$$

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30. ***REASONING*** Using either Equation 16.3 or 16.4 with $x = 0$ m, we obtain

$$y = A\sin\left(2\pi f t\right)$$

Applying this equation with $f = 175$ Hz, we can find the times at which $y = 0.10$ m.

SOLUTION Using the given values for y and A, we obtain

$$0.10\ \text{m} = \left(0.20\ \text{m}\right)\sin\left(2\pi f t\right) \qquad \text{or} \qquad 2\pi f t = \sin^{-1}\left(\frac{0.10\ \text{m}}{0.20\ \text{m}}\right) = \sin^{-1}\left(0.50\right)$$

The smallest two angles for which the sine function is 0.50 are 30° and 150°. However, the values for $2\pi f t$ must be expressed in radians. Thus, we find that the smallest two angles for which the sine function is 0.50 are expressed in radians as follows:

$$\left(30^\circ\right)\left(\frac{2\pi}{360^\circ}\right) \qquad \text{and} \qquad \left(150^\circ\right)\left(\frac{2\pi}{360^\circ}\right)$$

The times corresponding to these angles can be obtained as follows:

$$2\pi f t_1 = \left(30^\circ\right)\left(\frac{2\pi}{360^\circ}\right) \qquad 2\pi f t_2 = \left(150^\circ\right)\left(\frac{2\pi}{360^\circ}\right)$$

$$t_1 = \frac{30^\circ}{f\left(360^\circ\right)} \qquad t_2 = \frac{150^\circ}{f\left(360^\circ\right)}$$

Subtracting and using $f = 175$ Hz gives

$$t_2 - t_1 = \frac{150^\circ}{f\left(360^\circ\right)} - \frac{30^\circ}{f\left(360^\circ\right)} = \frac{\left(150^\circ - 30^\circ\right)}{\left(175\ \text{Hz}\right)\left(360^\circ\right)} = \boxed{1.9\times10^{-3}\ \text{s}}$$
