6. ***REASONING*** In Drawing 1 the two speakers are equidistant from the observer O. Since each wave travels the same distance in reaching the observer, the difference in travel-distances is zero, and constructive interference will occur for any frequency. Different frequencies will correspond to different wavelengths, but the path difference will always be zero. Condensations will always meet condensations and rarefactions will always meet rarefactions at the observation point. Since any frequency is acceptable in Drawing 1, our solution will focus on Drawing 2.

In Drawing 2, destructive interference occurs only when the difference in travel distances for the two waves is an odd integer number n of half-wavelengths. Only certain frequencies, therefore, will be consistent with this requirement.

SOLUTION The frequency f and wavelength λ are related by $\lambda = v/f$ (Equation 16.1), where v is the speed of the sound. Using this equation together with the requirement for destructive interference in drawing 2, we have

$$\underbrace{\sqrt{L^2+L^2}-L}_{\text{Difference in travel distances}} = n\frac{\lambda}{2} = n\frac{v}{2f} \qquad \text{where } n = 1,\ 3,\ 5,\ \ldots$$

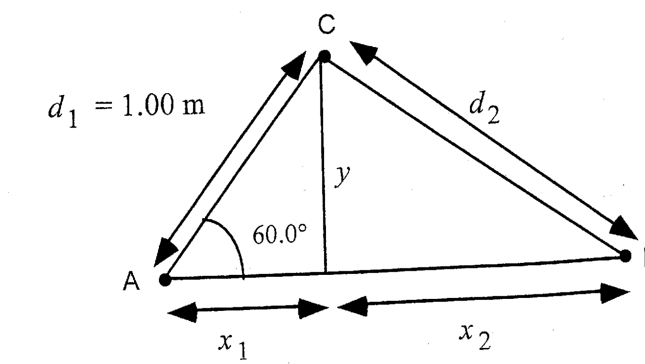
Here we have used the Pythagorean theorem to determine the length of the diagonal of the square. Solving for the frequency f gives

$$f = \frac{nv}{2\left(\sqrt{2}-1\right)L}$$

The problem asks for the minimum frequency, so we choose $n = 1$ and obtain

$$f = \frac{v}{2\left(\sqrt{2}-1\right)L} = \frac{343\ \text{m/s}}{2\left(\sqrt{2}-1\right)(0.75\ \text{m})} = \boxed{550\ \text{Hz}}$$

7. SSM WWW ***REASONING*** The geometry of the positions of the loudspeakers and the listener is shown in the following drawing.



The listener at C will hear either a loud sound or no sound, depending upon whether the interference occurring at C is constructive or destructive. If the listener hears no sound, destructive interference occurs, so

$$d_2 - d_1 = \frac{n\lambda}{2} \qquad n = 1, 3, 5, \ldots \qquad (1)$$

SOLUTION Since $v = \lambda f$, according to Equation 16.1, the wavelength of the tone is

$$\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{68.6\ \text{Hz}} = 5.00\ \text{m}$$

Speaker B will be closest to Speaker A when $n = 1$ in Equation (1) above, so

$$d_2 = \frac{n\lambda}{2} + d_1 = \frac{5.00\ \text{m}}{2} + 1.00\ \text{m} = 3.50\ \text{m}$$

From the figure above we have that,

$$x_1 = (1.00\ \text{m})\cos 60.0° = 0.500\ \text{m}$$

$$y = (1.00\ \text{m})\sin 60.0° = 0.866\ \text{m}$$

Then

$$x_2^2 + y^2 = d_2^2 = (3.50\ \text{m})^2 \qquad \text{or} \qquad x_2 = \sqrt{(3.50\ \text{m})^2 - (0.866\ \text{m})^2} = 3.39\ \text{m}$$

Therefore, the closest that speaker A can be to speaker B so that the listener hears no sound is $x_1 + x_2 = 0.500\ \text{m} + 3.39\ \text{m} = \boxed{3.89\ \text{m}}$.

8. ***REASONING*** The two speakers are vibrating exactly out of phase. This means that the conditions for constructive and destructive interference are opposite of those that apply when the speakers vibrate in phase, as they do in Example 1 in the text. Thus, for two wave sources vibrating exactly out of phase, a difference in path lengths that is zero or an integer number (1, 2, 3, …) of wavelengths leads to destructive interference; a difference in path lengths that is a half-integer number $\left(\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, \ldots\right)$ of wavelengths leads to constructive interference. First, we will determine the wavelength being produced by the speakers. Then, we will determine the difference in path lengths between the speakers and the observer and compare the differences to the wavelength in order to decide which type of interference occurs.

SOLUTION According to Equation 16.1, the wavelength λ is related to the speed v and frequency f of the sound as follows:

$$\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{429\ \text{Hz}} = 0.800\ \text{m}$$

Since ABC in Figure 17.7 is a right triangle, the Pythagorean theorem applies and the difference Δd in the path lengths is given by

$$\Delta d = d_{AC} - d_{BC} = \sqrt{d_{AB}^2 + d_{BC}^2} - d_{BC}$$

We will now apply this expression for parts (a) and (b).

a. When $d_{BC} = 1.15\ \text{m}$, we have

$$\Delta d = \sqrt{d_{AB}^2 + d_{BC}^2} - d_{BC} = \sqrt{(2.50\ \text{m})^2 + (1.15\ \text{m})^2} - 1.15\ \text{m} = 1.60\ \text{m}$$

Since $1.60\ \text{m} = 2(0.800\ \text{m}) = 2\lambda$, it follows that the interference is $\boxed{\text{destructive}}$ (the speakers vibrate out of phase).

b. When $d_{BC} = 2.00\ \text{m}$, we have

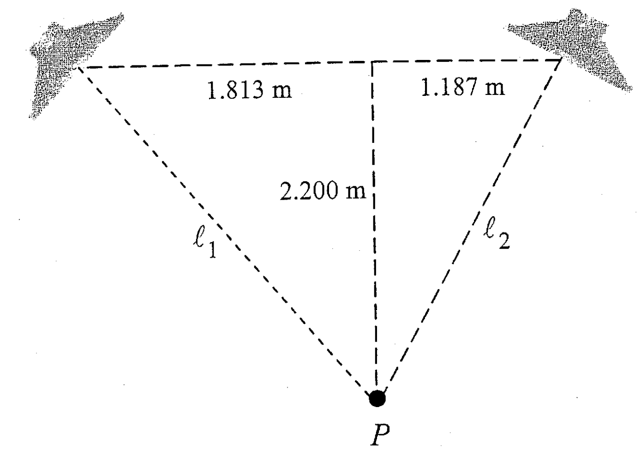
$$\Delta d = \sqrt{d_{AB}^2 + d_{BC}^2} - d_{BC} = \sqrt{(2.50\ \text{m})^2 + (2.00\ \text{m})^2} - 2.00\ \text{m} = 1.20\ \text{m}$$

Since $1.20\ \text{m} = 1.5(0.800\ \text{m}) = \left(1\frac{1}{2}\right)\lambda$, it follows that the interference is $\boxed{\text{constructive}}$ (the speakers vibrate out of phase).

9. ***REASONING*** When the difference $\ell_1 - \ell_2$ in path lengths traveled by the two sound waves is a half-integer number $\left(\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, \ldots\right)$ of wavelengths, destructive interference occurs at the listener. When the difference in path lengths is zero or an integer number (1, 2, 3, …) of wavelengths, constructive interference occurs. Therefore, we will divide the distance $\ell_1 - \ell_2$ by the wavelength of the sound to determine if constructive or destructive interference occurs. The wavelength is, according to Equation 16.1, the speed v of sound divided by the frequency f; $\lambda = v/f$.

SOLUTION

a. The distances ℓ_1 and ℓ_2 can be determined by applying the Pythagorean theorem to the two right triangles in the drawing:



$$\ell_1 = \sqrt{(2.200\ \text{m})^2 + (1.813\ \text{m})^2} = 2.851\ \text{m}$$

$$\ell_2 = \sqrt{(2.200\ \text{m})^2 + (1.187\ \text{m})^2} = 2.500\ \text{m}$$

Therefore, $\ell_1 - \ell_2 = 0.351\ \text{m}$. The wavelength of the sound is $\lambda = \dfrac{v}{f} = \dfrac{343\ \text{m/s}}{1466\ \text{Hz}} = 0.234\ \text{m}$. Dividing the distance $\ell_1 - \ell_2$ by the wavelength λ gives the number of wavelengths in this distance:

$$\text{Number of wavelengths} = \frac{\ell_1 - \ell_2}{\lambda} = \frac{0.351\ \text{m}}{0.233\ \text{m}} = 1.5$$

Since the number of wavelengths is a half-integer number $\left(1\frac{1}{2}\right)$, $\boxed{\text{destructive interference}}$ occurs at the listener.

b. The wavelength of the sound is now $\lambda = \dfrac{v}{f} = \dfrac{343\ \text{m/s}}{977\ \text{Hz}} = 0.351\ \text{m}$. Dividing the distance $\ell_1 - \ell_2$ by the wavelength λ gives the number of wavelengths in that distance:

$$\text{Number of wavelengths} = \frac{\ell_1 - \ell_2}{\lambda} = \frac{0.351\ \text{m}}{0.351\ \text{m}} = 1$$

Since the number of wavelengths is an integer number (1), $\boxed{\text{constructive interference}}$ occurs at the listener.

22. SSM **REASONING AND SOLUTION** The first case requires that the frequency be either

$$440\text{ Hz} - 5\text{ Hz} = 435\text{ Hz} \quad \text{or} \quad 440\text{ Hz} + 5\text{ Hz} = 445\text{ Hz}$$

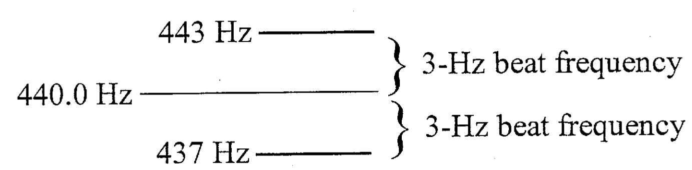
The second case requires that the frequency be either

$$436\text{ Hz} - 9\text{ Hz} = 427\text{ Hz} \quad \text{or} \quad 436\text{ Hz} + 9\text{ Hz} = 445\text{ Hz}$$

The frequency of the tuning fork is $\boxed{445\text{ Hz}}$.

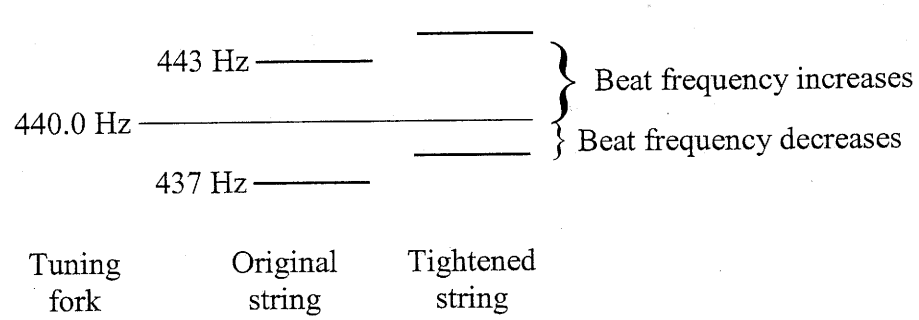
---

23. ***REASONING*** The beat frequency is the difference between two sound frequencies. Therefore, the original frequency of the guitar string (before it was tightened) was either 3 Hz lower than that of the tuning fork (440.0 Hz – 3 Hz = 337 Hz) or 3 Hz higher (440.0 Hz + 3 Hz = 443 Hz):



To determine which of these frequencies is the correct one (437 or 443 Hz), we will use the information that the beat frequency decreases when the guitar string is tightened

SOLUTION When the guitar string is tightened, its frequency of vibration (either 437 or 443 Hz) increases. As the drawing below shows, when the 437-Hz frequency increases, it becomes closer to 440.0 Hz, so the beat frequency decreases. When the 443-Hz frequency increases, it becomes farther from 440.0 Hz, so the beat frequency increases. Since the problem states that the beat frequency decreases, the original frequency of the guitar string was $\boxed{437\text{ Hz}}$.



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24. ***REASONING*** The beat frequency heard by the bystander is the difference between the frequency of the sound that the bystander hears from the moving car and that from the (stationary) parked car. The bystander hears a frequency f_o from the moving horn that is greater than the emitted frequency f_s. This is because of the Doppler effect (Section 16.9).

The bystander is the observer of the sound wave emitted by the horn. Since the horn is moving toward the observer, more condensations and rarefactions of the wave arrive at the observer's ear per second than would otherwise be the case. More cycles per second means that the observed frequency is greater than the emitted frequency.

The bystander also hears a frequency from the horn of the stationary car that is equal to the frequency f_s produced by the horn. Since this horn is stationary, there is no Doppler effect.

SOLUTION According to Equation 16.11, the frequency f_o that the bystander hears from the moving horn is

$$f_o = f_s\left(\frac{1}{1-\dfrac{v_s}{v}}\right)$$

where f_s is the frequency of the sound emitted by the horn, v_s is the speed of the moving horn, and v is the speed of sound. The beat frequency heard by the bystander is $f_o - f_s$, so we find that

$$f_o - f_s = f_s\left(\frac{1}{1-\dfrac{v_s}{v}}\right) - f_s = f_s\left(\frac{1}{1-\dfrac{v_s}{v}} - 1\right)$$

$$= (395\text{ Hz})\left(\frac{1}{1-\dfrac{12.0\text{ m/s}}{343\text{ m/s}}} - 1\right) = \boxed{14\text{ Hz}}$$

---

25. ***REASONING*** When the wave created by the tuning fork is superposed on the sound wave traveling through the seawater, the beat frequency f_{beat} heard by the underwater swimmer is equal to the difference between the two frequencies:

$$f_{\text{beat}} = f_{\text{wave}} - f_{\text{fork}} \tag{1}$$

We know the frequency f_{fork} of the tuning fork, and we will determine the frequency f_{wave} of the sound wave from its wavelength λ and speed v via $v = f_{\text{wave}}\lambda$ (Equation 16.1). The speed of a sound wave in a liquid is given by $v = \sqrt{\dfrac{B_{\text{ad}}}{\rho}}$ (Equation 16.6), where B_{ad} is the adiabatic bulk modulus of the liquid, and ρ is its density.

SOLUTION Solving $v = f_{\text{wave}}\lambda$ (Equation 16.1) for f_{wave} yields $f_{\text{wave}} = \dfrac{v}{\lambda}$. Substituting this result into Equation (1), we obtain

29. SSM *REASONING* The fundamental frequency f_1 is given by Equation 17.3 with $n = 1$: $f_1 = v/(2L)$. Since values for f_1 and L are given in the problem statement, we can use this expression to find the speed of the waves on the cello string. Once the speed is known, the tension F in the cello string can be found by using Equation 16.2, $v = \sqrt{F/(m/L)}$.

SOLUTION Combining Equations 17.3 and 16.2 yields

$$2Lf_1 = \sqrt{\frac{F}{m/L}}$$

Solving for F, we find that the tension in the cello string is

$$F = 4L^2 f_1^2 (m/L) = 4(0.800\text{ m})^2(65.4\text{ Hz})^2(1.56\times10^{-2}\text{ kg/m}) = \boxed{171\text{ N}}$$

30. *REASONING* The harmonic frequencies are integer multiples of the fundamental frequency. Therefore, for wire A (on which there is a second-harmonic standing wave), the fundamental frequency is one half of 660 Hz, or 330 Hz. Similarly, for wire B (on which there is a third-harmonic standing wave), the fundamental frequency is one third of 660 Hz, or 220 Hz. The fundamental frequency f_1 is related to the length L of the wire and the speed v at which individual waves travel back and forth on the wire by $f_1 = v/(2L)$ (Equation 17.3, with $n = 1$). This relation will allow us to determine the speed of the wave on each wire.

SOLUTION Using Equation 17.3 with $n = 1$, we find

$$f_1 = \frac{v}{2L} \quad \text{or} \quad v = 2Lf_1$$

Wire A $\quad v = 2(1.2\text{ m})(330\text{ Hz}) = \boxed{790\text{ m/s}}$

Wire B $\quad v = 2(1.2\text{ m})(220\text{ Hz}) = \boxed{530\text{ m/s}}$

31. SSM *REASONING* For standing waves on a string that is clamped at both ends, Equations 17.3 and 16.2 indicate that the standing wave frequencies are

$$f_n = n\left(\frac{v}{2L}\right) \quad \text{where} \quad v = \sqrt{\frac{F}{m/L}}$$

Combining these two expressions, we have, with $n = 1$ for the fundamental frequency,

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{m/L}}$$

This expression can be used to find the ratio of the two fundamental frequencies.

SOLUTION The ratio of the two fundamental frequencies is

$$\frac{f_{\text{old}}}{f_{\text{new}}} = \frac{\frac{1}{2L}\sqrt{\frac{F_{\text{old}}}{m/L}}}{\frac{1}{2L}\sqrt{\frac{F_{\text{new}}}{m/L}}} = \sqrt{\frac{F_{\text{old}}}{F_{\text{new}}}}$$

Since $F_{\text{new}} = 4F_{\text{old}}$, we have

$$f_{\text{new}} = f_{\text{old}}\sqrt{\frac{F_{\text{new}}}{F_{\text{old}}}} = f_{\text{old}}\sqrt{\frac{4F_{\text{old}}}{F_{\text{old}}}} = f_{\text{old}}\sqrt{4} = (55.0\text{ Hz})(2) = \boxed{1.10\times10^2\text{ Hz}}$$

32. *REASONING* Equation 17.3 (with $n = 1$) gives the fundamental frequency as $f_1 = v/(2L)$, where L is the wire's length and v is the wave speed on the wire. The speed is given by Equation 16.2 as $v = \sqrt{\frac{F}{m/L}}$, where F is the tension and m is the mass of the wire.

SOLUTION Using Equations 17.3 and 16.2, we obtain

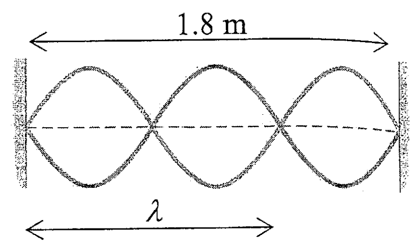
$$f_1 = \frac{v}{2L} = \frac{\sqrt{\frac{F}{m/L}}}{2L} = \frac{1}{2}\sqrt{\frac{F}{mL}} = \frac{1}{2}\sqrt{\frac{160\text{ N}}{(6.0\times10^{-3}\text{ kg})(0.41\text{ m})}} = \boxed{130\text{ Hz}}$$

33. *REASONING* A standing wave is composed of two oppositely traveling waves. The speed v of these waves is given by $v = \sqrt{\frac{F}{m/L}}$ (Equation 16.2), where F is the tension in the string and m/L is its linear density (mass per unit length). Both F and m/L are given in the statement of the problem. The wavelength λ of the waves can be obtained by visually inspecting the standing wave pattern. The frequency of the waves is related to the speed of the waves and their wavelength by $f = v/\lambda$ (Equation 16.1).

SOLUTION
a. The speed of the waves is

$$v=\sqrt{\frac{F}{m/L}}=\sqrt{\frac{280\ \text{N}}{8.5\times10^{-3}\,\text{kg/m}}}=\boxed{180\ \text{m/s}}$$

b. Two loops of any standing wave comprise one wavelength. Since the string is 1.8 m long and consists of three loops (see the drawing), the wavelength is



$$\lambda=\tfrac{2}{3}(1.8\ \text{m})=\boxed{1.2\ \text{m}}$$

c. The frequency of the waves is

$$f=\frac{v}{\lambda}=\frac{180\ \text{m/s}}{1.2\ \text{m}}=\boxed{150\ \text{Hz}}$$

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34. ***REASONING*** The series of natural frequencies for a wire fixed at both ends is given by $f_n=nv/(2L)$ (Equation 17.3), where the harmonic number n takes on the integer values 1, 2, 3, etc.. This equation can be solved for n. The natural frequency f_n is the lowest frequency that the human ear can detect, and the length L of the wire is given. The speed v at which waves travel on the wire can be obtained from the given values for the tension and the wire's linear density.

SOLUTION According to Equation 17.3, the harmonic number n is

$$n=\frac{f_n 2L}{v} \tag{1}$$

The speed v is $v=\sqrt{\frac{F}{m/L}}$ (Equation 16.2), where F is the tension and m/L is the linear density. Substituting this expression into Equation (1) gives

$$n=\frac{f_n 2L}{v}=\frac{f_n 2L}{\sqrt{\frac{F}{m/L}}}=f_n 2L\sqrt{\frac{m/L}{F}}=(20.0\ \text{Hz})2(7.60\ \text{m})\sqrt{\frac{0.0140\ \text{kg/m}}{323\ \text{N}}}=\boxed{2}$$

---

35. ***CONCEPT QUESTIONS*** The fundamental frequency f_1 of the wire is given by $f_1=v/(2L)$ (Equation 17.3, with $n=1$), where v is the speed at which the waves travel on the wire and L is the length of the wire. The speed is related to the tension F in the wire according to $v=\sqrt{\frac{F}{m/L}}$ (Equation 16.2), where m/L is the mass per unit length of the wire.

The tension in the wire in Part 2 of the drawing is less than the tension in Part 1. The reason is related to Archimedes' principle (see Equation 11.6). This principle indicates that when an object is immersed in a fluid, the fluid exerts an upward buoyant force on the object. In Part 2 the upward buoyant force from the mercury supports part of the block's weight, thus reducing the amount of the weight that the wire must support.

SOLUTION Substituting Equation 16.2 into Equation 17.3, we can obtain the fundamental frequency of the wire:

$$f_1=\frac{v}{2L}=\frac{1}{2L}\sqrt{\frac{F}{m/L}} \tag{1}$$

In Part 1 of the drawing, the tension F balances the weight of the block, keeping it from falling. The weight of the block is its mass times the acceleration due to gravity (see Equation 4.5). The mass, according to Equation 11.1 is the density ρ_{copper} times the volume V of the block. Thus, the tension in Part 1 is

Part 1 tension $\qquad F=(mass)\,g=\rho_{\text{copper}}Vg$

In Part 2 of the drawing, the tension is reduced from this amount by the amount of the upward buoyant force. According to Archimedes' principle, the buoyant force is the weight of the liquid mercury displaced by the block. Since half of the block's volume is immersed, the volume of mercury displaced is $\frac{1}{2}V$. The weight of this mercury is the mass times the acceleration due to gravity. Once again, according to Equation 11.1, the mass is the density ρ_{mercury} times the volume, which is $\frac{1}{2}V$. Thus, the tension in Part 2 is

Part 2 tension $\qquad F=\rho_{\text{copper}}Vg-\rho_{\text{mercury}}\left(\tfrac{1}{2}V\right)g$

With these two values for the tension we can apply Equation (1) to both parts of the drawing and obtain

Part 1 $\qquad f_1=\frac{1}{2L}\sqrt{\frac{\rho_{\text{copper}}Vg}{m/L}}$

Part 2 $\qquad f_1=\frac{1}{2L}\sqrt{\frac{\rho_{\text{copper}}Vg-\rho_{\text{mercury}}\left(\frac{1}{2}V\right)g}{m/L}}$

Dividing the fundamental frequency of Part 2 by that of Part 1 gives

$$\frac{f_{1,\text{ Part 2}}}{f_{1,\text{ Part 1}}} = \frac{\dfrac{1}{2L}\sqrt{\dfrac{\rho_{\text{copper}}Vg - \rho_{\text{mercury}}\left(\frac{1}{2}V\right)g}{m/L}}}{\dfrac{1}{2L}\sqrt{\dfrac{\rho_{\text{copper}}Vg}{m/L}}} = \sqrt{\frac{\rho_{\text{copper}} - \frac{1}{2}\rho_{\text{mercury}}}{\rho_{\text{copper}}}}$$

$$= \sqrt{\frac{8890\text{ kg/m}^3 - \frac{1}{2}\left(13\ 600\text{ kg/m}^3\right)}{8890\text{ kg/m}^3}} = \boxed{0.485}$$

---

36. ***REASONING*** Each string has a node at each end, so the frequency of vibration is given by Equation 17.3 as $f_n = nv/(2L)$, where $n = 1, 2, 3, \ldots$ The speed v of the wave can be determined from Equation 16.2 as $v = \sqrt{F/(m/L)}$. We will use these two relations to find the lowest frequency that permits standing waves in both strings with a node at the junction.

SOLUTION
Since the frequency of the left string is equal to the frequency of the right string, we can write

$$\frac{n_{\text{left}}\sqrt{\dfrac{F}{(m/L)_{\text{left}}}}}{2L_{\text{left}}} = \frac{n_{\text{right}}\sqrt{\dfrac{F}{(m/L)_{\text{right}}}}}{2L_{\text{right}}}$$

Substituting in the data given in the problem yields

$$\frac{n_{\text{left}}\sqrt{\dfrac{190.0\text{ N}}{6.00\times10^{-2}\text{ kg/m}}}}{2(3.75\text{ m})} = \frac{n_{\text{right}}\sqrt{\dfrac{190.0\text{ N}}{1.50\times10^{-2}\text{ kg/m}}}}{2(1.25\text{ m})}$$

This expression gives $n_{\text{left}} = 6n_{\text{right}}$. Letting $n_{\text{left}} = 6$ and $n_{\text{right}} = 1$, the frequency of the left string (which is also equal to the frequency of the right string) is

$$f_6 = \frac{(6)\sqrt{\dfrac{190.0\text{ N}}{6.00\times10^{-2}\text{ kg/m}}}}{2(3.75\text{ m})} = \boxed{45.0\text{ Hz}}$$

---

37. SSM ***REASONING*** We can find the extra length that the D-tuner adds to the E-string by calculating the length of the D-string and then subtracting from it the length of the E string. For standing waves on a string that is fixed at both ends, Equation 17.3 gives the frequencies

as $f_n = n(v/2L)$. The ratio of the fundamental frequency of the D-string to that of the E-string is

$$\frac{f_D}{f_E} = \frac{v/(2L_D)}{v/(2L_E)} = \frac{L_E}{L_D}$$

This expression can be solved for the length L_D of the D-string in terms of quantities given in the problem statement.

SOLUTION The length of the D-string is

$$L_D = L_E\left(\frac{f_E}{f_D}\right) = (0.628\text{ m})\left(\frac{41.2\text{ Hz}}{36.7\text{ Hz}}\right) = 0.705\text{ m}$$

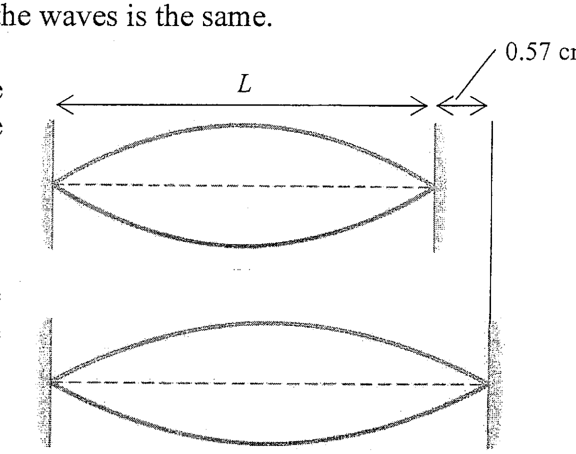
The length of the E-string is extended by the D-tuner by an amount

$$L_D - L_E = 0.705\text{ m} - 0.628\text{ m} = \boxed{0.077\text{ m}}$$

---

38. ***REASONING*** The beat frequency is equal to the higher frequency of the shorter string minus the lower frequency of the longer string. The reason the longer string has the lower frequency can be seen from the drawing, where it is evident that both strings are vibrating at their fundamental frequencies. The fundamental frequency of vibration (n = 1) for a string fixed at each end is given by $f_1 = v/(2L)$ (Equation 17.3). Since the speed v is the same for both strings (see the following paragraph), but the length L is greater for the longer string, the longer string vibrates at the lower frequency.

The waves on the longer string have the same speed as those on the shorter string. The speed v of a transverse wave on a string is given by $v = \sqrt{F/(m/L)}$ (Equation 16.2), where F is the tension in the string and m/L is the mass per unit length (or linear density). Since F and m/L are the same for both strings, the speed of the waves is the same.



SOLUTION The beat frequency is the frequency of the shorter string minus the frequency of the longer string; $f_{\text{shorter}} - f_{\text{longer}}$. We are given that $f_{\text{shorter}} = 225$ Hz.

According to Equation 17.3 with $n = 1$, we have $f_{\text{longer}} = v/(2L_{\text{longer}})$, where L_{longer} is the length of the longer string. According to the drawing, we have $L_{\text{longer}} = L + 0.0057$ m. Thus,

$$L_6 - L_7 = \frac{v}{2f_6} - \frac{v}{2f_7}$$

$$= \frac{v}{2\left(\sqrt[12]{2}\right)^6 f_0} - \frac{v}{2\left(\sqrt[12]{2}\right)^7 f_0} = \left(\frac{v}{2f_0}\right)\left[\frac{1}{\left(\sqrt[12]{2}\right)^6} - \frac{1}{\left(\sqrt[12]{2}\right)^7}\right]$$

$$= L_0\left[\frac{1}{\left(\sqrt[12]{2}\right)^6} - \frac{1}{\left(\sqrt[12]{2}\right)^7}\right] = (0.628 \text{ m})(0.0397) = \boxed{0.0249 \text{ m}}$$

---

41. SSM ***REASONING AND SOLUTION*** The distance between one node and an adjacent antinode is $\lambda/4$. Thus, we must first determine the wavelength of the standing wave. A tube open at only one end can develop standing waves only at the odd harmonic frequencies. Thus, for a tube of length L producing sound at the third harmonic (n = 3), $L = 3(\lambda/4)$. Therefore, the wavelength of the standing wave is

$$\lambda = \tfrac{4}{3}L = \tfrac{4}{3}(1.5 \text{ m}) = 2.0 \text{ m}$$

and the distance between one node and the adjacent antinode is $\lambda/4 = \boxed{0.50 \text{ m}}$.

---

42. ***REASONING*** Equation 17.5 (with n = 1) gives the fundamental frequency as $f_1 = v/(4L)$, where L is the length of the auditory canal and v is the speed of sound.

SOLUTION Using Equation 17.5, we obtain

$$f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(0.029 \text{ m})} = \boxed{3.0\times10^3 \text{ Hz}}$$

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43. ***REASONING*** The frequency of a pipe open at both ends is given by Equation 17.4 as $f_n = n\left(\frac{v}{2L}\right)$, where n is an integer specifying the harmonic number, v is the speed of sound, and L is the length of the pipe.

SOLUTION Solving the equation above for L, and recognizing that n = 3 for the third harmonic, we have

$$L = n\left(\frac{v}{2f_n}\right) = 3\left[\frac{343 \text{ m/s}}{2(262 \text{ Hz})}\right] = \boxed{1.96 \text{ m}}$$

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44. ***REASONING*** The speed v of sound is related to its frequency f and wavelength λ by $v = f\lambda$ (Equation 16.1). At the end of the tube where the tuning fork is, there is an antinode, because the gas molecules there are free to vibrate. At the plunger, there is a node, because the gas molecules there are not free to vibrate. Since there is an antinode at one end of the tube and a node at the other, the smallest value of L occurs when the length of the tube is one quarter of a wavelength.

SOLUTION Since the smallest value for L is a quarter of a wavelength, we have $L = \frac{1}{4}\lambda$ or $\lambda = 4L$. According to Equation 16.1, the speed of sound is

$$v = f\lambda = f(4L) = (485 \text{ Hz})(4\times0.264 \text{ m}) = \boxed{512 \text{ m/s}}$$

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45. ***REASONING*** The fundamental frequency f_1^{A} of air column A, which is open at both ends, is given by Equation 17.4 with n = 1: $f_1^{\text{A}} = (1)\left(\frac{v}{2L_{\text{A}}}\right)$, where v is the speed of sound in air and L_{A} is the length of the air column. Similarly, the fundamental frequency f_1^{B} of air column B, which is open at only one end, can be expressed using Equation 17.5 with n = 1: $f_1^{\text{B}} = (1)\left(\frac{v}{4L_{\text{B}}}\right)$. These two relations will allow us to determine the length of air column B.

SOLUTION Since the fundamental frequencies of the two air columns are the same $f_1^{\text{A}} = f_1^{\text{B}}$, so that

$$\underbrace{(1)\left(\frac{v}{2L_{\text{A}}}\right)}_{f_1^{\text{A}}} = \underbrace{(1)\left(\frac{v}{4L_{\text{B}}}\right)}_{f_1^{\text{B}}} \quad \text{or} \quad L_{\text{B}} = \tfrac{1}{2}L_{\text{A}} = \tfrac{1}{2}(0.70 \text{ m}) = \boxed{0.35 \text{ m}}$$

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46. ***REASONING*** For a tube open at only one end, the series of natural frequencies is given by $f_n = nv/(4L)$ (Equation 17.5), where n has the values 1, 3, 5, etc., v is the speed of sound, and L is the tube length. We will apply this expression to both the air-filled and the helium-filled tube in order to determine the desired ratio.

SOLUTION According to Equation 17.5, we have

$$f_{n,\,\text{air}} = \frac{nv_{\text{air}}}{4L} \quad \text{and} \quad f_{n,\,\text{helium}} = \frac{nv_{\text{helium}}}{4L}$$