CHAPTER 21 | MAGNETIC FORCES AND MAGNETIC FIELDS

PROBLEMS

1. SSM ***REASONING AND SOLUTION*** The magnitude of the force can be determined using Equation 21.1, $F = |q| vB \sin\theta$, where θ is the angle between the velocity and the magnetic field. The direction of the force is determined by using Right-Hand Rule No. 1.

a. $F = |q| vB \sin 30.0° = (8.4 \times 10^{-6}\ \text{C})(45\ \text{m/s})(0.30\ \text{T}) \sin 30.0° = \boxed{5.7 \times 10^{-5}\ \text{N}}$, directed $\boxed{\text{into the paper}}$.

b. $F = |q| vB \sin 90.0° = (8.4 \times 10^{-6}\ \text{C})(45\ \text{m/s})(0.30\ \text{T}) \sin 90.0° = \boxed{1.1 \times 10^{-4}\ \text{N}}$, directed $\boxed{\text{into the paper}}$.

c. $F = |q| vB \sin 150° = (8.4 \times 10^{-6}\ \text{C})(45\ \text{m/s})(0.30\ \text{T}) \sin 150° = \boxed{5.7 \times 10^{-5}\ \text{N}}$, directed $\boxed{\text{into the paper}}$.

2. ***REASONING*** The electron's acceleration is related to the net force ΣF acting on it by Newton's second law: $a = \Sigma F/m$ (Equation 4.1), where m is the electron's mass. Since we are ignoring the gravitational force, the net force is that caused by the magnetic force, whose magnitude is expressed by Equation 21.1 as $F = |q_0| vB \sin\theta$. Thus, the magnitude of the electron's acceleration can be written as $a = \left(|q_0| vB \sin\theta\right)/m$.

SOLUTION We note that $\theta = 90.0°$, since the velocity of the electron is perpendicular to the magnetic field. The magnitude of the electron's charge is 1.60×10^{-19} C, and the electron's mass is 9.11×10^{-31} kg (see the inside of the front cover), so

$$a = \frac{|q_0| vB \sin\theta}{m}$$

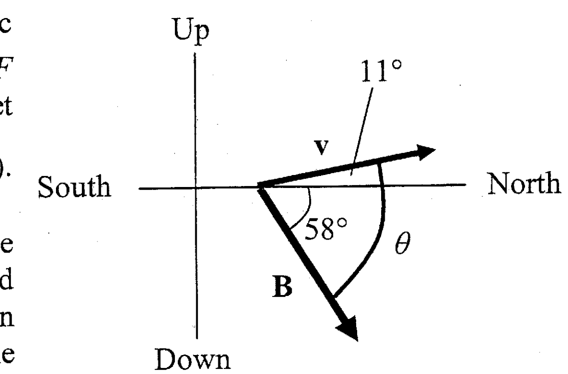
$$= \frac{\left(1.60 \times 10^{-19}\ \text{C}\right)\left(2.1 \times 10^{6}\ \text{m/s}\right)\left(1.6 \times 10^{-5}\ \text{T}\right)\sin 90.0°}{9.11 \times 10^{-31}\ \text{kg}} = \boxed{5.9 \times 10^{12}\ \text{m/s}^2}$$

3. SSM ***REASONING*** According to Equation 21.1, the magnitude of the magnetic force on a moving charge is $F = |q_0| vB \sin\theta$. Since the magnetic field points due north and the proton moves eastward, $\theta = 90.0°$. Furthermore, since the magnetic force on the moving proton balances its weight, we have $mg = |q_0| vB \sin\theta$, where m is the mass of the proton. This expression can be solved for the speed v.

SOLUTION Solving for the speed v, we have

$$v = \frac{mg}{|q_0| B \sin\theta} = \frac{(1.67 \times 10^{-27}\ \text{kg})(9.80\ \text{m/s}^2)}{(1.6 \times 10^{-19}\ \text{C})(2.5 \times 10^{-5}\ \text{T}) \sin 90.0°} = \boxed{4.1 \times 10^{-3}\ \text{m/s}}$$

4. ***REASONING*** The magnitude $|q_0|$ of the electric charge of the bullet is related to the magnitude F of the magnetic force exerted on the bullet according to $B = \dfrac{F}{|q_0|(v \sin\theta)}$ (Equation 21.1). Here, θ is the angle between the direction of the bullet's velocity **v** and the earth's magnetic field **B**. (See the drawing, which depicts the situation as seen from the side.) We will find the magnitude of the bullet's charge from Equation 21.1, and Right-Hand Rule No. 1 will allow us to determine the algebraic sign of the charge.



SOLUTION Solving $B = \dfrac{F}{|q_0|(v \sin\theta)}$ (Equation 21.1) for $|q_0|$, we obtain

$$|q_0| = \frac{F}{B(v \sin\theta)} \tag{1}$$

The angle θ is the angle between the directions of the bullet's velocity **v** and the direction of earth's magnetic field **B**. Therefore, θ is the sum of the angles that these vectors make with the horizontal (see the drawing):

$$\theta = 58° + 11° = 69°$$

Applying Right-Hand Rule No. 1 to the vectors **v** and **B** (see the drawing), we see that the force on a positively charged bullet would point into the page, which is west. Because the force on this bullet points to the east, out of the page, the bullet's charge must be negative.

When removing the absolute value brackets from Equation (1), therefore, we must insert a minus sign on the right hand side. Making this change, we find that

$$q_0 = -\frac{F}{B(v\sin\theta)} = -\frac{2.8\times10^{-10}\ \text{N}}{\left(5.4\times10^{-5}\ \text{T}\right)(670\ \text{m/s})\sin 69^\circ} = \boxed{-8.3\times10^{-9}\ \text{C}}$$

5. ***REASONING*** According to Equation 21.1, the magnetic force has a magnitude of $F = |q|\,vB\sin\theta$, where $|q|$ is the magnitude of the charge, B is the magnitude of the magnetic field, v is the speed, and θ is the angle of the velocity with respect to the field. As θ increases from 0° to 90°, the force increases, so the angle must lie between 25° and 90°.

SOLUTION Letting $\theta_1 = 25°$ and θ_2 be the desired angle, we can apply Equation 21.1 to both situations as follows:

$$\underbrace{F = |q|\,vB\sin\theta_1}_{\text{Situation 1}} \quad \text{and} \quad \underbrace{2F = |q|\,vB\sin\theta_2}_{\text{Situation 2}}$$

Dividing the equation for situation 2 by the equation for situation 1 gives

$$\frac{2F}{F} = \frac{|q|\,vB\sin\theta_2}{|q|\,vB\sin\theta_1} \quad \text{or} \quad \sin\theta_2 = 2\sin\theta_1 = 2\sin 25° = 0.85$$

$$\theta_2 = \sin^{-1}(0.85) = \boxed{58°}$$

6. ***REASONING*** A moving charge experiences no magnetic force when its velocity points in the direction of the magnetic field or in the direction opposite to the magnetic field. Thus, the magnetic field must point either in the direction of the $+x$ axis or in the direction of the $-x$ axis. If a moving charge experiences the maximum possible magnetic force when moving in a magnetic field, then the velocity must be perpendicular to the field. In other words, the angle θ that the charge's velocity makes with respect to the magnetic field is $\theta = 90°$.

SOLUTION The magnitude B of the magnetic field can be determined using Equation 21.1:

$$B = \frac{F}{|q|\,v\sin\theta} = \frac{0.48\ \text{N}}{\left(8.2\times10^{-6}\ \text{C}\right)\left(5.0\times10^{5}\ \text{m/s}\right)\sin 90^\circ} = \boxed{0.12\ \text{T}}$$

In this calculation we use $\theta = 90°$, because the 0.48-N force is the maximum possible force. Since the particle experiences no magnetic force when it moves along the $+x$ axis, we can conclude that the magnetic field points

$\boxed{\text{either in the direction of the } +x \text{ axis or in the direction of the } -x \text{ axis}}$.

7. ***REASONING*** When a charge q_0 travels at a speed v and its velocity makes an angle θ with respect to a magnetic field of magnitude B, the magnetic force acting on the charge has a magnitude F that is given by $F = |q_0|\,vB\sin\theta$ (Equation 21.1). We will solve this problem by applying this expression twice, first to the motion of the charge when it moves perpendicular to the field so that $\theta = 90.0°$ and then to the motion when $\theta = 38°$.

SOLUTION When the charge moves perpendicular to the field so that $\theta = 90.0°$, Equation 21.1 indicates that

$$F_{90.0°} = |q_0|\,vB\sin 90.0°$$

When the charge moves so that $\theta = 38°$, Equation 21.1 shows that

$$F_{38°} = |q_0|\,vB\sin 38°$$

Dividing the second expression by the first expression gives

$$\frac{F_{38°}}{F_{90.0°}} = \frac{|q_0|\,vB\sin 38°}{|q_0|\,vB\sin 90.0°}$$

$$F_{38°} = F_{90.0°}\left(\frac{\sin 38°}{\sin 90.0°}\right) = \left(2.7\times10^{-3}\ \text{N}\right)\left(\frac{\sin 38°}{\sin 90.0°}\right) = \boxed{1.7\times10^{-3}\ \text{N}}$$

8. ***REASONING*** According to Equation 21.1, the magnetic force has a magnitude of $F = |q|\,vB\sin\theta$. The field B and the directional angle θ are the same for each particle. Particle 1, however, travels faster than particle 2. By itself, a faster speed v would lead to a greater force magnitude F. But the force on each particle is the same. Therefore, particle 1 must have a smaller charge to counteract the effect of its greater speed.

SOLUTION Applying Equation 21.1 to each particle, we have

$$\underbrace{F = |q_1|\,v_1 B\sin\theta}_{\text{Particle 1}} \quad \text{and} \quad \underbrace{F = |q_2|\,v_2 B\sin\theta}_{\text{Particle 2}}$$

Dividing the equation for particle 1 by the equation for particle 2 and remembering that $v_1 = 3v_2$ gives

14. ***REASONING*** The time t that it takes the particle to complete one revolution is the time … travel a distance $d = 2\pi r$ equal to the circumference of a circle of radius r at a speed v. From Equation 2.1, we know that speed is the ratio of distance to elapsed time $\left(v = \frac{d}{t}\right)$, so the elapsed time is the ratio of distance to speed:

$$t = \frac{d}{v} = \frac{2\pi r}{v}$$

Because the particle follows a circular path that is perpendicular to the external magnetic field of magnitude B, the radius of the path is given by $r = \frac{mv}{|q|B}$ (Equation 21.2), where m is the mass and $|q|$ is the magnitude of the charge of the particle. We will use Equation 21.2 to determine the speed of the particle, and then Equation (1) to find the time for one complete revolution.

SOLUTION Solving $r = \frac{mv}{|q|B}$ (Equation 21.2) for v yields

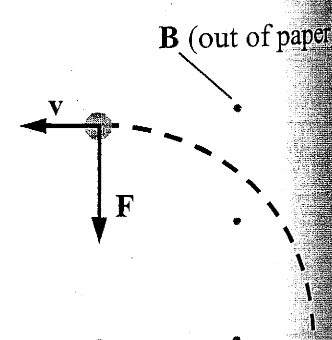
$$v = \frac{|q|Br}{m} = \frac{|q|}{m}Br$$

In the last step of Equation (2), we have expressed the speed v explicitly in terms of the charge-to-mass ratio $|q|/m$ of the particle. Substituting Equation (2) into Equation (1), we obtain

$$t = \frac{2\pi r}{v} = \frac{2\pi \cancel{r}}{\frac{|q|}{m}B\cancel{r}} = \frac{2\pi}{\frac{|q|}{m}B} = \frac{2\pi}{\left(5.7\times10^{8}\ \text{C/kg}\right)\left(0.72\ \text{T}\right)} = \boxed{1.5\times10^{-8}\ \text{s}}$$

15. ***REASONING***

a. The drawing shows the velocity **v** of the particle at the top of its path. The magnetic force **F**, which provides the centripetal force, must be directed toward the center of the circular path. Since the directions of **v**, **F**, and **B** are known, we can use Right-Hand Rule No. 1 (RHR-1) to determine if the charge is positive or negative.



b. The radius of the circular path followed by a charged particle is given by Equation 21.2 as $r = mv/|q|B$. The mass m of the particle can be obtained directly from this relation, since all other variables are known.

SOLUTION

a. If the particle were positively charged, an application of RHR-1 would show that the force would be directed straight up, opposite to that shown in the drawing. Thus, the charge on the particle must be $\boxed{\text{negative}}$.

b. Solving Equation 21.2 for the mass of the particle gives

$$m = \frac{|q|Br}{v} = \frac{\left(8.2\times10^{-4}\ \text{C}\right)\left(0.48\ \text{T}\right)\left(960\ \text{m}\right)}{140\ \text{m/s}} = \boxed{2.7\times10^{-3}\ \text{kg}}$$

16. ***REASONING*** Equation 21.2 gives the radius r of the circular path as $r = mv/(|q|B)$, where m, v, and $|q|$ are, respectively, the mass, speed, and charge magnitude of the particle, and B is the magnitude of the magnetic field. We wish the radius to be the same for both the proton and the electron. The speed v and the charge magnitude $|q|$ are the same for the proton and the electron, but the mass of the electron is 9.11×10^{-31} kg, while that of the proton is 1.67×10^{-27} kg. Therefore, to offset the effect of the smaller electron mass m in Equation 21.2, the magnitude B of the field must be reduced for the electron.

SOLUTION Applying Equation 21.2 to the proton and the electron, both of which carry charges of the same magnitude $|q| = e$, we obtain

$$\underbrace{r = \frac{m_p v}{eB_p}}_{\text{Proton}} \quad \text{and} \quad \underbrace{r = \frac{m_e v}{eB_e}}_{\text{Electron}}$$

Dividing the proton-equation by the electron-equation gives

$$\frac{r}{r} = \frac{\frac{m_p v}{eB_p}}{\frac{m_e v}{eB_e}} \quad \text{or} \quad 1 = \frac{m_p B_e}{m_e B_p}$$

Solving for B_e, we obtain

$$B_e = \frac{m_e B_p}{m_p} = \frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(0.50\ \text{T}\right)}{1.67\times10^{-27}\ \text{kg}} = \boxed{2.7\times10^{-4}\ \text{T}}$$

$$r = \frac{mv}{|q|B} = \frac{m}{|q|B}\sqrt{\frac{2|q|V}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{|q|}}$$

Using e to denote the magnitude of the charge on an electron, we note that the charge for species X^+ is $+e$, while the charge for species X^{2+} is $+2e$. With this in mind, we find for the ratio of the radii that

$$\frac{r_1}{r_2} = \frac{\frac{1}{B}\sqrt{\frac{2mV}{e}}}{\frac{1}{B}\sqrt{\frac{2mV}{2e}}} = \sqrt{2} = \boxed{1.41}$$

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23. SSM ***REASONING*** When the proton moves in the magnetic field, its trajectory is a circular path. The proton will just miss the opposite plate if the distance between the plates is equal to the radius of the path. The radius is given by Equation 21.2 as $r = mv/(|q|B)$. This relation can be used to find the magnitude B of the magnetic field, since values for all the other variables are known.

SOLUTION Solving the relation $r = mv/(|q|B)$ for the magnitude of the magnetic field, and realizing that the radius is equal to the plate separation, we find that

$$B = \frac{mv}{|q|r} = \frac{(1.67 \times 10^{-27}\text{ kg})(3.5\times10^{6}\text{ m/s})}{(1.60 \times 10^{-19}\text{ C})(0.23\text{ m})} = \boxed{0.16\text{ T}}$$

The values for the mass and the magnitude of the charge (which is the same as that of the electron) have been taken from the inside of the front cover.

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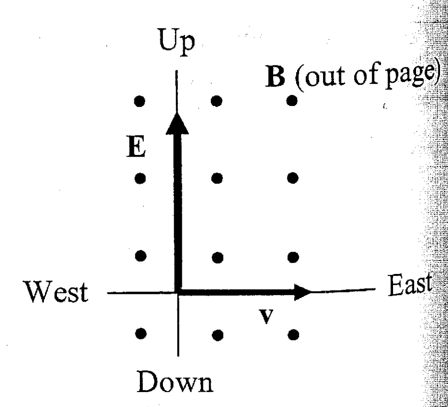
24. ***REASONING*** The magnitude F_B of the magnetic force acting on the particle is related to its speed v by $F_B = |q_0| v B \sin\theta$ (Equation 21.1), where B is the magnitude of the magnetic field, q_0 is the particle's charge, and θ is the angle between the magnetic field **B** and the particle's velocity **v**. As the drawing shows, the vector **v** (east, to the right) is perpendicular to the vector **B** (south, out of the page). Therefore, $\theta = 90°$, and Equation 21.1 becomes



$$F_B = |q_0| v B \sin 90° = |q_0| v B \tag{1}$$

In addition to the magnetic force, there is also an electric force of magnitude F_E acting on the particle. This force magnitude does not depend upon the speed v of the particle, as we see from $F_E = |q_0| E$ (Equation 18.2). The particle is positively charged, so the electric force acting on it points upward in the same direction as the electric field. By Right-Hand Rule No.1, the magnetic force acting on the positively charged particle points down, and is therefore opposite to the electric force. The net force on the particle points upward, so we conclude that the electric force is greater than the magnetic force. Thus, the magnitude F of the net force acting on the particle is equal to the magnitude of the electric force minus the magnitude of the magnetic force:

$$F = F_E - F_B \tag{2}$$

We also note that particles traveling at a speed $v_0 = 6.50\times10^3$ m/s experience no net force. Therefore, $F_E = F_B$ for particles moving at the speed v_0.

SOLUTION Substituting Equation (1) and $F_E = |q_0| E$ (Equation 18.2) into Equation (2) yields

$$F = |q_0| E - |q_0| v B \tag{3}$$

The magnetic field magnitude B is not given, but, as noted above, for particles with speed $v_0 = 6.50\times10^3$ m/s, the magnetic force of Equation (1), $F_B = |q_0| v_0 B$, is equal to the electric force $F_E = |q_0| E$ (Equation 18.2). Therefore, we have that

$$\cancel{|q_0|} v_0 B = \cancel{|q_0|} E \quad \text{or} \quad B = \frac{E}{v_0} \tag{4}$$

Substituting Equation (4) into Equation (3) yields

$$F = |q_0| E - |q_0| v B = |q_0| E - \frac{|q_0| v E}{v_0} \tag{5}$$

Solving Equation (5) for v, we obtain

$$\frac{|q_0| v E}{v_0} = |q_0| E - F \quad \text{or} \quad \frac{v}{v_0} = 1 - \frac{F}{|q_0| E} \quad \text{or} \quad v = v_0\left(1 - \frac{F}{|q_0| E}\right) \tag{6}$$

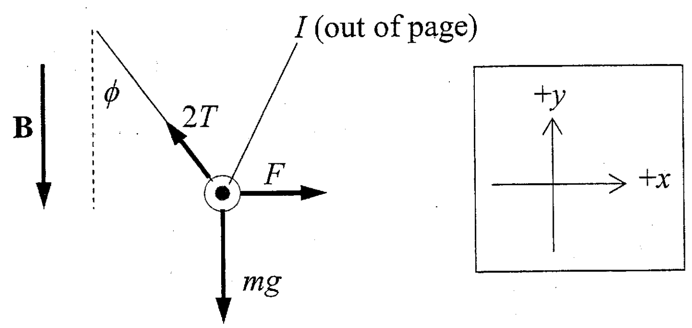
Substituting the given values into Equation (6), we find that

$$F = ILB\sin\theta = IL\sqrt{B_x^2 + B_y^2}\ \sin\theta$$

$$= (4.3\text{ A})(0.25\text{ m})\sqrt{(0.10\text{ T})^2 + (0.15\text{ T})^2}\ \sin 90° = \boxed{0.19\text{ N}}$$

38. ***REASONING***
There are four forces that act on the wire: the magnetic force (magnitude F), the weight mg of the wire, and the tension in each of the two strings (magnitude T in each string). Since there are two strings, the following drawing shows the total tension as $2T$. The magnitude F of the magnetic force is given by $F = ILB \sin\theta$ (Equation 21.3), where I is the current, L is the length of the wire, B is the magnitude of the magnetic field, and θ is the angle between the wire and the magnetic field. In this problem $\theta = 90.0°$.

The direction of the magnetic force is given by Right-Hand Rule No. 1 (see Section 21.5). The drawing shows an end view of the wire, where it can be seen that the magnetic force (magnitude = F) points to the right, in the $+x$ direction.



In order for the wire to be in equilibrium, the net force ΣF_x in the x-direction must be zero, and the net force ΣF_y in the y-direction must be zero: $\Sigma F_x = 0$ (Equation 4.9a) and $\Sigma F_y = 0$ (Equation 4.9b). These equations will allow us to determine the angle ϕ and the tension T.

SOLUTION
Since the wire is in equilibrium, the sum of the forces in the x direction is zero:

$$\underbrace{-2T\sin\phi + F}_{\Sigma F_x} = 0$$

Substituting in $F = ILB \sin 90.0°$ for the magnitude of the magnetic force, this equation becomes

$$\underbrace{-2T\sin\phi + ILB\sin 90.0°}_{\Sigma F_x} = 0 \qquad (1)$$

The sum of the forces in the y direction is also zero:

$$\underbrace{+2T\cos\phi - mg}_{\Sigma F_y} = 0 \qquad (2)$$

Since these two equations contain two unknowns, ϕ and T, we can solve for each of them.

a. To obtain the angle ϕ, we solve Equation (2) for the tension $\left(T = \dfrac{mg}{2\cos\phi}\right)$ and substitute the result into Equation (1). This gives

$$-2\left(\frac{mg}{2\cos\phi}\right)\sin\phi + ILB\sin 90.0° = 0 \quad \text{or} \quad \underbrace{\frac{\sin\phi}{\cos\phi}}_{\tan\phi} = \frac{ILB}{mg}$$

Thus,

$$\phi = \tan^{-1}\left(\frac{ILB}{mg}\right) = \tan^{-1}\left[\frac{(42\text{ A})(0.20\text{ m})(0.070\text{ T})}{(0.080\text{ kg})(9.80\text{ m/s}^2)}\right] = \boxed{37°}$$

b. The tension in each wire can be found directly from Equation (2):

$$T = \frac{mg}{2\cos\phi} = \frac{(0.080\text{ kg})(9.80\text{ m/s}^2)}{2\cos 37°} = \boxed{0.49\text{ N}}$$

39. $\boxed{\text{SSM}}$ ***REASONING*** Since the rod does not rotate about the axis at P, the net torque relative to that axis must be zero; $\Sigma\tau = 0$ (Equation 9.2). There are two torques that must be considered, one due to the magnetic force and another due to the weight of the rod. We consider both of these to act at the rod's center of gravity, which is at the geometrical center of the rod (length = L), because the rod is uniform. According to Right-Hand Rule No. 1, the magnetic force acts perpendicular to the rod and is directed up and to the left in the drawing. Therefore, the magnetic torque is a counterclockwise (positive) torque. Equation 21.3 gives the magnitude F of the magnetic force as $F = ILB \sin 90.0°$, since the current is perpendicular to the magnetic field. The weight is mg and acts downward, producing a clockwise (negative) torque. The magnitude of each torque is the magnitude of the force times the lever arm (Equation 9.1). Thus, we have for the torques:

$$\tau_{\text{magnetic}} = +\underbrace{(ILB)}_{\text{force}}\underbrace{(L/2)}_{\text{lever arm}} \quad \text{and} \quad \tau_{\text{weight}} = -\underbrace{(mg)}_{\text{force}}\underbrace{[(L/2)\cos\theta]}_{\text{lever arm}}$$

Setting the sum of these torques equal to zero will enable us to find the angle θ that the rod makes with the ground.

SOLUTION Setting the sum of the torques equal to zero gives $\Sigma\tau = \tau_{\text{magnetic}} + \tau_{\text{weight}} = 0$, and we have

$$+(ILB)(L/2) - (mg)\left[(L/2)\cos\theta\right] = 0 \quad \text{or} \quad \cos\theta = \frac{ILB}{mg}$$

$$\theta = \cos^{-1}\left[\frac{(4.1\text{ A})(0.45\text{ m})(0.36\text{ T})}{(0.094\text{ kg})(9.80\text{ m/s}^2)}\right] = \boxed{44°}$$

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40. ***REASONING AND SOLUTION***
a. From Right-Hand Rule No. 1, if we extend the right hand so that the fingers point in the direction of the magnetic field, and the thumb points in the direction of the current, the palm of the hand faces the direction of the magnetic force on the current.

The springs will stretch when the magnetic force exerted on the copper rod is downward, toward the bottom of the page. Therefore, if you extend your right hand with your fingers pointing out of the page and the palm of your hand facing the bottom of the page, your thumb points left-to-right along the copper rod. Thus, the current flows $\boxed{\text{left-to-right}}$ in the copper rod.

b. The downward magnetic force exerted on the copper rod is, according to Equation 21.3

$$F = ILB\sin\theta = (12\text{ A})(0.85\text{ m})(0.16\text{ T})\sin 90.0° = 1.6\text{ N}$$

According to Equation 10.1, the force F_x^{Applied} required to stretch each spring is $F_x^{\text{Applied}} = kx$, where k is the spring constant. Since there are two springs, we know that the magnetic force F exerted on the current must equal $2F_x^{\text{Applied}}$, so that $F = 2F_x^{\text{Applied}} = 2kx$. Solving for x, we find that

$$x = \frac{F}{2k} = \frac{1.6\text{ N}}{2(75\text{ N/m})} = \boxed{1.1\times10^{-2}\text{ m}}$$

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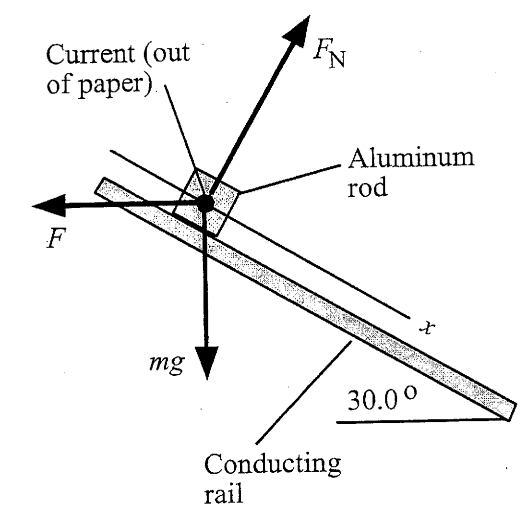
41. ***REASONING*** The following drawing shows a side view of the conducting rails and the aluminum rod. Three forces act on the rod: (1) its weight mg, (2) the magnetic force F, and the normal force F_N. An application of Right-Hand Rule No. 1 shows that the magnetic force is directed to the left, as shown in the drawing. Since the rod slides down the rails at a constant velocity, its acceleration is zero. If we choose the x-axis to be along the rails, Newton's second law states that the net force along the x-direction is zero: $\Sigma F_x = ma_x = 0$. Using the components of F and mg that are along the x-axis, Newton's second law becomes

$$\underbrace{-F\cos 30.0° + mg\sin 30.0°}_{\Sigma F_x} = 0$$

The magnetic force is given by Equation 21.3 as $F = ILB\sin\theta$, where $\theta = 90.0°$ is the angle between the magnetic field and the current. We can use these two relations to find the current in the rod.



SOLUTION Substituting the expression $F = ILB\sin 90.0°$ into Newton's second law and solving for the current I, we obtain

$$I = \frac{mg\sin 30.0°}{(LB\sin 90.0°)\cos 30.0°} = \frac{(0.20\text{ kg})(9.80\text{ m/s}^2)\sin 30.0°}{\left[(1.6\text{ m})(0.050\text{ T})\sin 90.0°\right]\cos 30.0°} = \boxed{14\text{ A}}$$

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42. ***REASONING*** According to Equation 21.4, the maximum torque is $\tau_{\text{max}} = NIAB$, where N is the number of turns in the coil, I is the current, $A = \pi r^2$ is the area of the circular coil, and B is the magnitude of the magnetic field. We can apply the maximum-torque expression to each coil, noting that τ_{max}, N, and I are the same for each.

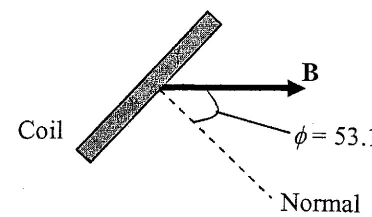
SOLUTION Applying Equation 21.4 to each coil, we have

$$\underbrace{\tau_{\text{max}} = NI\pi r_1^2 B_1}_{\text{Coil 1}} \quad \text{and} \quad \underbrace{\tau_{\text{max}} = NI\pi r_2^2 B_2}_{\text{Coil 2}}$$

Dividing the expression for coil 2 by the expression for coil 1 gives

$$\frac{\tau_{\text{max}}}{\tau_{\text{max}}} = \frac{NI\pi r_2^2 B_2}{NI\pi r_1^2 B_1} \quad \text{or} \quad 1 = \frac{r_2^2 B_2}{r_1^2 B_1}$$

b. The edge-on view of the coil at the right shows the normal to the plane of the coil, the magnetic field **B**, and the angle ϕ.



47. SSM WWW ***REASONING*** The torque on the loop is given by Equation 21.4 $\tau = NIAB\sin\phi$. From the drawing in the text, we see that the angle ϕ between the normal to the plane of the loop and the magnetic field is $90° - 35° = 55°$. The area of the loop is $0.70\ \text{m} \times 0.50\ \text{m} = 0.35\ \text{m}^2$.

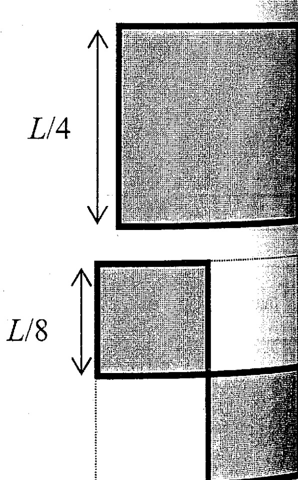
SOLUTION

a. The magnitude of the net torque exerted on the loop is

$$\tau = NIAB\sin\phi = (75)(4.4\ \text{A})(0.35\ \text{m}^2)(1.8\ \text{T})\ \sin 55° = \boxed{170\ \text{N}\cdot\text{m}}$$

b. As discussed in the text, when a current-carrying loop is placed in a magnetic field, the loop tends to rotate such that its normal becomes aligned with the magnetic field. The normal to the loop makes an angle of 55° with respect to the magnetic field. Since this angle decreases as the loop rotates, the $\boxed{35°\ \text{angle increases}}$.

48. ***REASONING*** When the wire is used to make a single-turn square coil, each side of the square has a length of $\frac{1}{4}L$. When the wire is used to make a two-turn coil, each side of the square has a length of $\frac{1}{8}L$. The drawing shows these two options and indicates that the total effective area of NA is greater for the single-turn option. Hence, more torque is obtained by using the single-turn option.



SOLUTION According to Equation 21.4 the maximum torque experienced by the coil is $\tau_{max} = (NIAB)\sin 90° = NIAB$, where N is the number of turns, I is the current, A is the area of each turn, and B is the magnitude of the magnetic field. Applying this expression to each option gives

Single-turn
$$\tau_{max} = NIAB = NI\left(\tfrac{1}{4}L\right)^2 B$$
$$= (1)(1.7\ \text{A})\left[\tfrac{1}{4}(1.00\ \text{m})\right]^2(0.34\ \text{T}) = \boxed{0.036\ \text{N}\cdot\text{m}}$$

Two-turn
$$\tau_{max} = NIAB = NI\left(\tfrac{1}{8}L\right)^2 B$$
$$= (2)(1.7\ \text{A})\left[\tfrac{1}{8}(1.00\ \text{m})\right]^2(0.34\ \text{T}) = \boxed{0.018\ \text{N}\cdot\text{m}}$$

49. ***REASONING*** The magnitude τ of the torque that acts on a current-carrying coil placed in a magnetic field is given by $\tau = NIAB\sin\phi$ (Equation 21.4), where N is the number of loops in the coil ($N = 1$ in this problem), I is the current, A is the area of one loop, B is the magnitude of the magnetic field (the same for each coil), and ϕ is the angle (the same for each coil) between the normal to the coil and the magnetic field. Since we are given that the torque for the square coil is the same as that for the circular coil, we can write

$$\underbrace{(1)I_{square}A_{square}B\sin\phi}_{\tau_{square}} = \underbrace{(1)I_{circle}A_{circle}B\sin\phi}_{\tau_{circle}}$$

This relation can be used directly to find the ratio of the currents.

SOLUTION Solving the equation above for the ratio of the currents yields

$$\frac{I_{square}}{I_{circle}} = \frac{A_{circle}}{A_{square}}$$

If the length of each wire is L, the length of each side of the square is $\frac{1}{4}L$, and the area of the square coil is $A_{square} = \left(\frac{1}{4}L\right)\left(\frac{1}{4}L\right) = \frac{1}{16}L^2$. The area of the circular coil is $A_{circle} = \pi r^2$, where r is the radius of the coil. Since the circumference $(2\pi r)$ of the circular coil is equal to the length L of the wire, we have $2\pi r = L$, or $r = L/(2\pi)$. Substituting this value for r into the expression for the area of the circular coil gives $A_{circle} = \pi\left[L/(2\pi)\right]^2$. Thus, the ratio of the currents is

$$\frac{I_{square}}{I_{circle}} = \frac{A_{circle}}{A_{square}} = \frac{\pi\left(\frac{L}{2\pi}\right)^2}{\frac{1}{16}L^2} = \frac{4}{\pi} = \boxed{1.27}$$

77. SSM **_REASONING AND SOLUTION_** The current associated with the lightning bolt is

$$I = \frac{\Delta q}{\Delta t} = \frac{15\ \text{C}}{1.5\times10^{-3}\ \text{s}} = 1.0\times10^{4}\ \text{A}$$

The magnetic field near this current is given by

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.0\times10^{4}\ \text{A}\right)}{2\pi(25\ \text{m})} = \boxed{8.0\times10^{-5}\ \text{T}}$$

78. **_REASONING_** The magnitude of the magnetic force exerted on a long straight wire is given by Equation 21. 3 as $F = ILB \sin\theta$. The direction of the magnetic force is predicted by Right-Hand Rule No. 1. The net force on the triangular loop is the vector sum of the forces on the three sides.

SOLUTION

a. The direction of the current in side AB is opposite to the direction of the magnetic field, so the angle θ between them is $\theta = 180°$. The magnitude of the magnetic force is

$$F_{AB} = ILB\sin\theta = ILB\sin 180° = \boxed{0\ \text{N}}$$

For the side BC, the angle is $\theta = 55.0°$, and the length of the side is

$$L = \frac{2.00\ \text{m}}{\cos 55.0°} = 3.49\ \text{m}$$

The magnetic force is

$$F_{BC} = ILB\sin\theta = (4.70\ \text{A})(3.49\ \text{m})(1.80\ \text{T})\sin 55.0° = \boxed{24.2\ \text{N}}$$

An application of Right-Hand No. 1 shows that the magnetic force on side BC is directed $\boxed{\text{perpendicularly out of the paper}}$, toward the reader.

For the side AC, the angle is $\theta = 90.0°$. We see that the length of the side is

$$L = (2.00\ \text{m})\tan 55.0° = 2.86\ \text{m}$$

The magnetic force is

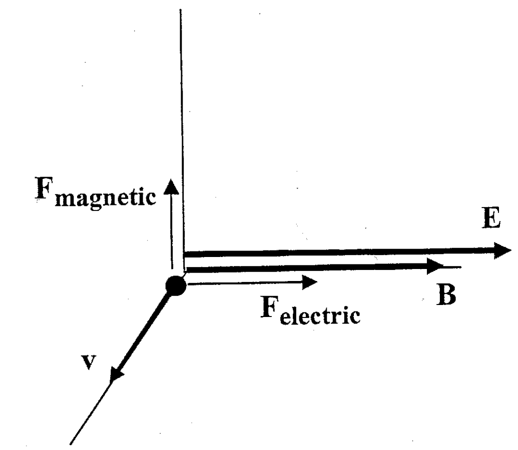
$$F_{AC} = ILB\sin\theta = (4.70\ \text{A})(2.86\ \text{m})(1.80\ \text{T})\sin 90.0° = \boxed{24.2\ \text{N}}$$

An application of Right-Hand No. 1 shows that the magnetic force on side AC is directed $\boxed{\text{perpendicularly into the paper}}$, away from the reader.

b. The net force is the vector sum of the forces on the three sides. Taking the positive direction as being out of the paper, the net force is

$$\sum F = 0\ \text{N} + 24.2\ \text{N} + (-24.2\ \text{N}) = \boxed{0\ \text{N}}$$

79. **_REASONING_** The magnetic field applies the maximum magnetic force to the moving charge, because the motion is perpendicular to the field. This force is perpendicular to both the field and the velocity. The electric field applies an electric force to the charge that is in the same direction as the field, since the charge is positive. These two forces are shown in the drawing, and they are perpendicular to one another. Therefore, the magnitude of the net field can be obtained using the Pythagorean theorem.



SOLUTION According to Equation 21.1, the magnetic force has a magnitude of $F_{\text{magnetic}} = |q| vB \sin\theta$, where $|q|$ is the magnitude of the charge, B is the magnitude of the magnetic field, v is the speed, and $\theta = 90°$ is the angle of the velocity with respect to the field. Thus, $F_{\text{magnetic}} = |q| vB$. According to Equation 18.2, the electric force has a magnitude of $F_{\text{electric}} = |q| E$. Using the Pythagorean theorem, we find the magnitude of the net force to be

$$F = \sqrt{F_{\text{magnetic}}^2 + F_{\text{electric}}^2} = \sqrt{\left(|q|vB\right)^2 + \left(|q|E\right)^2} = |q|\sqrt{(vB)^2 + E^2}$$

$$= \left(1.8\times10^{-6}\ \text{C}\right)\sqrt{\left[\left(3.1\times10^{6}\ \text{m/s}\right)\left(1.2\times10^{-3}\ \text{T}\right)\right]^2 + \left(4.6\times10^{3}\ \text{N/C}\right)^2} = \boxed{1.1\times10^{-2}\ \text{N}}$$

80. **_REASONING_** The two rods attract each other because they each carry a current I in the same direction. The bottom rod floats because it is in equilibrium. The two forces that act on the bottom rod are the downward force of gravity mg and the upward magnetic force of attraction to the upper rod. If the two rods are a distance s apart, the magnetic field generated by the top rod at the location of the bottom rod is (see Equation 21.5) $B = \mu_0 I/(2\pi s)$. According to Equation 21.3, the magnetic force exerted on the bottom rod is $F = ILB\sin\theta = \mu_0 I^2 L \sin\theta/(2\pi s)$, where θ is the angle between the magnetic field at