12. (d) Since $I_{rms} = V_{rms}/Z$ (Equation 23.6), the current is a maximum when the impedance Z is a minimum. The impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$ (Equation 23.7), and it has a minimum value when $X_C = X_L = 50\ \Omega$.

13. (c) The inductor has a very small reactance at low frequencies and behaves as if it were replaced by a wire with no resistance. Therefore, the circuit behaves as two resistors, R_1 and R_2, connected in parallel. The inductor has a very large reactance at high frequencies and behaves as if it were cut out of the circuit, leaving a gap in the connecting wires. The circuit behaves as a single resistance R_2 connected across the generator. The situation at low frequency gives rise to the largest possible current, because the effective resistance of the parallel combination is smaller than the resistance R_2.

14. (a) The capacitor has a very small reactance at high frequencies and behaves as if it were replaced by a wire with no resistance. Therefore, the circuit behaves as two resistors, R_1 and R_2, connected in parallel. The capacitor has a very large reactance at low frequencies and behaves as if it were cut out of the circuit, leaving a gap in the connecting wires. Therefore, the circuit behaves as a single resistor R_1 connected across the generator. The situation at high frequencies gives rise to the largest possible current, because the effective resistance of the parallel combination is smaller than the resistance R_1.

15. (e) At low frequencies, the capacitor has a very large reactance. In the series circuit, this large reactance gives rise to a large impedance and, hence, a small current. The parallel circuit has the larger current, because current can flow through the inductor, which has a small reactance at low frequencies.

16. (b) The resonant frequency f_0 is given by $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ (Equation 23.10). It depends only on C and L, and not on R.

17. $f_0 = 1.3 \times 10^3$ Hz

18. (d) The resonant frequency f_0 is given by $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ (Equation 23.10). When a second capacitor is added in parallel, the equivalent capacitance increases (see Section 20.12). Therefore, the resonant frequency decreases.

CHAPTER 23 | ALTERNATING CURRENT CIRCUITS

PROBLEMS

1. SSM ***REASONING*** The voltage across the capacitor reaches its maximum instantaneous value when the generator voltage reaches its maximum instantaneous value. The maximum value of the capacitor voltage first occurs one-fourth of the way, or one-quarter of a period, through a complete cycle (see the voltage curve in Figure 23.4).

SOLUTION The period of the generator is $T = 1/f = 1/(5.00\text{ Hz}) = 0.200\text{ s}$. Therefore, the least amount of time that passes before the instantaneous voltage across the capacitor reaches its maximum value is $\frac{1}{4}T = \frac{1}{4}(0.200\text{ s}) = \boxed{5.00\times10^{-2}\text{ s}}$.

2. ***REASONING*** When two capacitors of capacitance C are connected in parallel, their equivalent capacitance C_P is given by $C_P = C + C = 2C$ (Equation 20.18). We will find the capacitive reactance X_C of the equivalent capacitance from $X_C = \dfrac{1}{2\pi f C_P}$ (Equation 23.2). Because the capacitors are the only devices connected to the generator, the rms voltage V_{rms} across the capacitors is equal to the rms voltage of the generator. Therefore, the capacitive reactance X_C is related to the rms voltage of the generator and the rms current I_{rms} in the circuit by $V_{rms} = I_{rms}X_C$ (Equation 23.1).

SOLUTION Substituting $C_P = 2C$ into $X_C = \dfrac{1}{2\pi f C_P}$ (Equation 23.2) and solving for C, we obtain

$$X_C = \frac{1}{2\pi f C_P} = \frac{1}{2\pi f(2C)} = \frac{1}{4\pi f C} \quad \text{or} \quad C = \frac{1}{4\pi f X_C} \qquad (1)$$

Solving $V_{rms} = I_{rms}X_C$ (Equation 23.1) for X_C yields $X_C = \dfrac{V_{rms}}{I_{rms}}$. Substituting this result into Equation (1), we obtain

$$C = \frac{1}{4\pi f X_C} = \frac{1}{4\pi f\left(\dfrac{V_{rms}}{I_{rms}}\right)} = \frac{I_{rms}}{4\pi f V_{rms}} = \frac{0.16\text{ A}}{4\pi(610\text{ Hz})(24\text{ V})} = \boxed{8.7\times10^{-7}\text{ F}}$$

$$\underbrace{I_{1,\,\text{rms}}=\frac{V_{\text{rms}}}{X_{\text{L}}}=\frac{V_{\text{rms}}}{2\pi f L_1}}_{L_1\text{ alone}} \quad \text{and} \quad \underbrace{I_{2,\,\text{rms}}=\frac{V_{\text{rms}}}{X_{\text{L}}}=\frac{V_{\text{rms}}}{2\pi f L_2}}_{L_2\text{ alone}}$$

The current delivered to L_1 alone is

$$I_{1,\,\text{rms}}=\frac{V_{\text{rms}}}{2\pi f L_1}=\frac{240\text{ V}}{2\pi(2200\text{ Hz})(6.0\times10^{-3}\text{ H})}=\boxed{2.9\text{ A}}$$

The current delivered to the parallel combination of L_1 and L_2 is the sum of that delivered individually to each inductor and is

$$I_{\text{P, rms}}=I_{1,\,\text{rms}}+I_{2,\,\text{rms}}=\frac{V_{\text{rms}}}{2\pi f L_1}+\frac{V_{\text{rms}}}{2\pi f L_2}=\frac{V_{\text{rms}}}{2\pi f}\left(\frac{1}{L_1}+\frac{1}{L_2}\right)$$

$$=\frac{240\text{ V}}{2\pi(2200\text{ Hz})}\left(\frac{1}{6.0\times10^{-3}\text{ H}}+\frac{1}{9.0\times10^{-3}\text{ H}}\right)=\boxed{4.8\text{ A}}$$

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13. SSM WWW ***REASONING*** Since the capacitor and the inductor are connected in parallel, the voltage across each of these elements is the same or $V_{\text{L}}=V_{\text{C}}$. Using Equations 23.3 and 23.1, respectively, this becomes $I_{\text{rms}}X_{\text{L}}=I_{\text{rms}}X_{\text{C}}$. Since the currents in the inductor and capacitor are equal, this relation simplifies to $X_{\text{L}}=X_{\text{C}}$. Therefore, we can find the value of the inductance by equating the expressions (Equations 23.4 and 23.2) for the inductive reactance and the capacitive reactance, and solving for L.

SOLUTION Since $X_{\text{L}}=X_{\text{C}}$, we have

$$2\pi f L=\frac{1}{2\pi f C}$$

Therefore, the value of the inductance is

$$L=\frac{1}{4\pi^2 f^2 C}=\frac{1}{4\pi^2(60.0\text{ Hz})^2(40.0\times10^{-6}\text{ F})}=0.176\text{ H}=\boxed{176\text{ mH}}$$

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14. ***REASONING*** The inductance L of the inductor determines its inductive reactance X_{L} according to $X_{\text{L}}=2\pi fL$ (Equation 23.4), where f is the frequency of the generator. When the inductor is connected to the terminals of the generator, the rms voltage V_{rms} of the generator drives an rms current I_{rms} that depends upon the inductive reactance via

$X_{\text{L}}=\dfrac{V_{\text{rms}}}{I_{\text{rms}}}$ (Equation 23.3). We note that the generator frequency is given in kHz, where 1 kHz = 10^3 Hz, and the rms current is given in mA, where 1 mA = 10^{-3} A.

SOLUTION Solving $X_{\text{L}}=2\pi fL$ (Equation 23.4) for L yields

$$L=\frac{X_{\text{L}}}{2\pi f} \tag{2}$$

Substituting $X_{\text{L}}=\dfrac{V_{\text{rms}}}{I_{\text{rms}}}$ (Equation 23.3) into Equation (1), we find that

$$L=\frac{X_{\text{L}}}{2\pi f}=\frac{\left(\dfrac{V_{\text{rms}}}{I_{\text{rms}}}\right)}{2\pi f}=\frac{V_{\text{rms}}}{2\pi f I_{\text{rms}}}=\frac{39\text{ V}}{2\pi(7.5\times10^3\text{ Hz})(42\times10^{-3}\text{ A})}=\boxed{0.020\text{ H}}$$

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15. ***REASONING***

a. The inductive reactance X_{L} depends on the frequency f of the current and the inductance L through the relation $X_{\text{L}}=2\pi fL$ (Equation 23.4). This equation can be used directly to find the frequency of the current.

b. The capacitive reactance X_{C} depends on the frequency f of the current and the capacitance C through the relation $X_{\text{C}}=1/(2\pi fC)$ (Equation 23.2). By setting $X_{\text{C}}=X_{\text{L}}$ as specified in the problem statement, the capacitance can be found.

c. Since the inductive reactance is directly proportional to the frequency, tripling the frequency triples the inductive reactance.

d. The capacitive reactance is inversely proportional to the frequency, so tripling the frequency reduces the capacitive reactance by a factor of one-third.

SOLUTION

a. The frequency of the current is

$$f=\frac{X_{\text{L}}}{2\pi L}=\frac{2.10\times10^3\ \Omega}{2\pi\left(30.0\times10^{-3}\text{ H}\right)}=\boxed{1.11\times10^4\text{ Hz}} \tag{23.4}$$

b. The capacitance is

SOLUTION With only the resistor connected, Equation 20.14 indicates that the resistance is

$$R = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{112\ \text{V}}{0.500\ \text{A}} = 224\ \Omega$$

With only the inductor connected, Equation 23.3 indicates that the inductive reactance is

$$X_{\text{L}} = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{112\ \text{V}}{0.400\ \text{A}} = 2.80 \times 10^2\ \Omega$$

a. Using these values for R and X_{L} in Equations (1) and (2), we find that the impedance is

$$Z = \sqrt{R^2 + X_{\text{L}}^2} = \sqrt{(224\ \Omega)^2 + \left(2.80 \times 10^2\ \Omega\right)^2} = \boxed{359\ \Omega}$$

b. The phase angle ϕ between the current and the voltage of the generator is

$$\tan\phi = \frac{X_{\text{L}}}{R} \quad \text{or} \quad \phi = \tan^{-1}\left(\frac{X_{\text{L}}}{R}\right) = \tan^{-1}\left(\frac{2.80 \times 10^2\ \Omega}{224\ \Omega}\right) = \boxed{51.3^\circ}$$

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19. SSM ***REASONING*** The voltage supplied by the generator can be found from Equation 23.6, $V_{\text{rms}} = I_{\text{rms}} Z$. The value of I_{rms} is given in the problem statement, so we must obtain the impedance of the circuit.

SOLUTION The impedance of the circuit is, according to Equation 23.7,

$$Z = \sqrt{R^2 + (X_{\text{L}} - X_{\text{C}})^2} = \sqrt{(275\ \Omega)^2 + (648\ \Omega - 415\ \Omega)^2} = 3.60 \times 10^2\ \Omega$$

The rms voltage of the generator is

$$V_{\text{rms}} = I_{\text{rms}} Z = (0.233\ \text{A})(3.60 \times 10^2\ \Omega) = \boxed{83.9\ \text{V}}$$

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20. ***REASONING*** The phase angle is given by $\tan\phi = (X_{\text{L}} - X_{\text{C}})/R$ (Equation 23.8). When a series circuit contains only a resistor and a capacitor, the inductive reactance X_{L} is zero, and the phase angle is negative, signifying that the current leads the voltage of the generator. The impedance is given by Equation 23.7 with $X_{\text{L}} = 0\ \Omega$, or $Z = \sqrt{R^2 + X_{\text{C}}^2}$. Since the phase angle ϕ and the impedance Z are given, we can use these relations to find the resistance R and X_{C}.

SOLUTION Since the phase angle is negative, we can conclude that only a resistor and a capacitor are present. Using Equations 23.8, then, we have

$$\tan\phi = \frac{-X_{\text{C}}}{R} \quad \text{or} \quad X_{\text{C}} = -R\tan(-75.0^\circ) = 3.73R$$

According to Equation 23.7, the impedance is

$$Z = 192\ \Omega = \sqrt{R^2 + X_{\text{C}}^2}$$

Substituting $X_{\text{C}} = 3.73R$ into this expression for Z gives

$$192\ \Omega = \sqrt{R^2 + (3.73\ R)^2} = \sqrt{14.9\ \text{R}^2} \quad \text{or} \quad (192\ \Omega)^2 = 14.9\ \text{R}^2$$

$$R = \sqrt{\frac{(192\ \Omega)^2}{14.9}} = \boxed{49.7\ \Omega}$$

Using the fact that $X_{\text{C}} = 3.73\ R$, we obtain

$$X_{\text{C}} = 3.73\ (49.7\ \Omega) = \boxed{185\ \Omega}$$

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21. SSM ***REASONING*** We can use the equations for a series RCL circuit to solve this problem provided that we set $X_{\text{C}} = 0\ \Omega$ since there is no capacitor in the circuit. The current in the circuit can be found from Equation 23.6, $V_{\text{rms}} = I_{\text{rms}} Z$, once the impedance of the circuit has been obtained. Equation 23.8, $\tan\phi = (X_{\text{L}} - X_{\text{C}})/R$, can then be used (with $X_{\text{C}} = 0\ \Omega$) to find the phase angle between the current and the voltage.

SOLUTION The inductive reactance is (Equation 23.4)

$$X_{\text{L}} = 2\pi f L = 2\pi(106\ \text{Hz})(0.200\ \text{H}) = 133\ \Omega$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_{\text{L}} - X_{\text{C}})^2} = \sqrt{R^2 + X_{\text{L}}^2} = \sqrt{(215\ \Omega)^2 + (133\ \Omega)^2} = 253\ \Omega$$

a. The current through each circuit element is, using Equation 23.6,

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{234\text{ V}}{253\ \Omega} = \boxed{0.925\text{ A}}$$

b. The phase angle between the current and the voltage is, according to Equation 23.8 (with $X_C = 0\ \Omega$),

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{X_L}{R} = \frac{133\ \Omega}{215\ \Omega} = 0.619 \quad \text{or} \quad \phi = \tan^{-1}(0.619) = \boxed{31.8°}$$

22. ***REASONING*** The average power dissipated is that dissipated in the resistor and is $\overline{P} = I_{rms}^2 R$, according to Equation 20.15b. We are given the current I_{rms} but need to find the resistance R. Since the inductive reactance X_L is known, we can find the resistance from the impedance, which is $Z = \sqrt{R^2 + X_L^2}$, according to Equation 23.7. Since the voltage and the current are known, we can obtain the impedance from Equation 23.6 as $Z = V_{rms}/I_{rms}$.

SOLUTION From Equation 23.7, we can determine the resistance as $R = \sqrt{Z^2 - X_L^2}$. With this expression for the resistance, Equation 20.15b for the power becomes

$$\overline{P} = I_{rms}^2 R = I_{rms}^2\sqrt{Z^2 - X_L^2}$$

Using Equation 23.6 to express the impedance, we obtain the following value for the dissipated power

$$\overline{P} = I_{rms}^2\sqrt{Z^2 - X_L^2} = I_{rms}^2\sqrt{\left(\frac{V_{rms}}{I_{rms}}\right)^2 - X_L^2}$$

$$= (1.75\text{ A})^2\sqrt{\left(\frac{115\text{ V}}{1.75\text{ A}}\right)^2 - (52.0\ \Omega)^2} = \boxed{123\text{ W}}$$

23. ***REASONING AND SOLUTION*** In order to find the voltage and phase angle we need to know the impedance of the circuit. We know that

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\left(4.80\times10^3\text{ Hz}\right)\left(0.250\times10^{-6}\text{ F}\right)} = 133\ \Omega$$

The impedance of the circuit can be found using

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{(232\ \Omega)^2 + (0 - 133\ \Omega)^2} = 267\ \Omega$$

a. The voltage of the generator is

$$V = IZ = (0.0400\text{ A})(267\ \Omega) = \boxed{10.7\text{ V}}$$

b. The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 133\ \Omega}{232\ \Omega}\right) = \boxed{-29.8°}$$

REASONING The rms current I_{rms} in the circuit in part *c* of the drawing is equal to the rms voltage V_{rms} divided by the impedance Z or $I_{rms} = \frac{V_{rms}}{Z}$ (Equation 23.6). The impedance Z of a series RC circuit is given by Equation 23.7 with $X_L = 0\ \Omega$, since there is no inductance in the circuit; $Z = \sqrt{R^2 + X_C^2}$. The resistance R is known and the capacitive reactance X_C can be obtained from the relation $X_C = 1/(2\pi f C)$ (Equation 23.2), where C is the capacitance and f is the frequency. The capacitance, however, is related to the time constant τ of the circuit in part *a* of the drawing. The time constant of an RC circuit is the time for the capacitor to lose 63.2% of its initial charge (see the discussion in Section 20.13), and it is equal to the product of the resistance R and the capacitance C; $\tau = RC$ (Equation 20.21).

SOLUTION Substituting $Z = \sqrt{R^2 + X_C^2}$ into $I_{rms} = \frac{V_{rms}}{Z}$ gives

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + X_C^2}} \qquad (1)$$

Substituting $X_C = 1/(2\pi f C)$ (Equation 23.2) into Equation (1) allows us to write the rms current in the circuit in part *c* of the drawing as follows:

$$I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X_C^2}} = \frac{V_{rms}}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}}$$

Substituting $C = \tau/R$ (Equation 20.21) into Equation (1), we arrive at an expression for the rms current: