SOLUTION

a. The angle of refraction θ_2 is given by Snell's law, Equation 26.2, as

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left[\frac{(1.4)\sin 55°}{1.6}\right] = \boxed{46°}$$

b. The actual angle of refraction is

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left[\frac{(1.5)\sin 55°}{1.6}\right] = \boxed{50°}$$

c. The angle of refraction is

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left[\frac{(1.6)\sin 55°}{1.4}\right] = \boxed{69°}$$

d. The actual angle of refraction is

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left[\frac{(1.6)\sin 0°}{1.4}\right] = \boxed{0°}$$

---

11. ***REASONING*** The angle of refraction θ_2 is related to the angle of incidence θ_1 by Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (Equation 26.2), where n_1 and n_2 are, respectively the indices of refraction of the incident and refracting media. For each case (ice and water), the variables θ_1, n_1 and n_2, are known, so the angles of refraction can be determined.

SOLUTION The ray of light impinges from air (n_1 = 1.000) onto either the ice or water at an angle of incidence of θ_1 = 60.0°. Using n_2 = 1.309 for ice and n_2 = 1.333 for water, we find that the angles of refraction are

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2} \quad \text{or} \quad \theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)$$

Ice

$$\theta_{2,\,\text{ice}} = \sin^{-1}\left[\frac{(1.000)\sin 60.0°}{1.309}\right] = 41.4°$$

Water

$$\theta_{2,\,\text{water}} = \sin^{-1}\left[\frac{(1.000)\sin 60.0°}{1.333}\right] = 40.5°$$

The difference in the angles of refraction is $\theta_{2,\,\text{ice}} - \theta_{2,\,\text{water}} = 41.4° - 40.5° = \boxed{0.9°}$

…SONING AND SOLUTION The angle of incidence is found from the drawing to be

$$\theta_1 = \tan^{-1}\left(\frac{8.0\text{ m}}{2.5\text{ m}}\right) = 73°$$

…'s law gives the angle of refraction to be

$$\sin\theta_2 = (n_1/n_2)\sin\theta_1 = (1.000/1.333)\sin 73° = 0.72 \qquad \text{or} \qquad \theta_2 = 46°$$

The distance d is found from the drawing to be

$$d = 8.0\text{ m} + (4.0\text{ m})\tan\theta_2 = \boxed{12.1\text{ m}}$$

SSM ***REASONING*** We will use the geometry of the situation to determine the angle of incidence. Once the angle of incidence is known, we can use Snell's law to find the index of refraction of the unknown liquid. The speed of light v in the liquid can then be determined.

SOLUTION From the drawing in the text, we see that the angle of incidence at the liquid-air interface is

$$\theta_1 = \tan^{-1}\left(\frac{5.00\text{ cm}}{6.00\text{ cm}}\right) = 39.8°$$

The drawing also shows that the angle of refraction is 90.0°. Thus, according to Snell's law (Equation 26.2: $n_1 \sin\theta_1 = n_2 \sin\theta_2$), the index of refraction of the unknown liquid is

$$n_1 = \frac{n_2 \sin\theta_2}{\sin\theta_1} = \frac{(1.000)(\sin 90.0°)}{\sin 39.8°} = 1.56$$

From Equation 26.1 ($n = c/v$), we find that the speed of light in the unknown liquid is

$$v = \frac{c}{n_1} = \frac{3.00\times 10^8\text{ m/s}}{1.56} = \boxed{1.92\times 10^8\text{ m/s}}$$

REASONING The index of refraction of the oil is one of the factors that determine the apparent depth of the bolt. Equation 26.3 gives the apparent depth and can be solved for the index of refraction.

$$v = \frac{d}{t} \qquad \text{and} \qquad v' = \frac{d'}{t} \tag{1}$$

The apparent depth d' is given by $d' = d\left(\frac{n_2}{n_1}\right)$ (Equation 26.3), where $n_2 = 1.000$ is the index of refraction of air and $n_1 = 1.333$ (See Table 26.1) is the index of refraction of water.

SOLUTION Solving the first of Equations (1) for t yields $t = \frac{d}{v}$. Substituting this expression into the second of Equations (1), we obtain

$$v' = \frac{d'}{t} = \frac{d'}{\left(\frac{d}{v}\right)} = v\left(\frac{d'}{d}\right) \tag{2}$$

Substituting $d' = d\left(\frac{n_2}{n_1}\right)$ (Equation 26.3) into Equation (2), we find that

$$v' = v\left(\frac{d'}{d}\right) = v\frac{\cancel{d}\left(\frac{n_2}{n_1}\right)}{\cancel{d}} = v\left(\frac{n_2}{n_1}\right) = (0.48 \text{ m/s})\left(\frac{1.000}{1.333}\right) = \boxed{0.36 \text{ m/s}}$$

19. ***REASONING AND SOLUTION*** The horizontal distance of the chest from the normal is found from Figure 26.4*b* to be $x = d \tan \theta_1$ and $x = d' \tan \theta_2$, where θ_1 is the angle from the dashed normal to the solid rays and θ_2 is the angle from the dashed normal to the dashed rays. Hence,

$$d' = d\,(\tan \theta_1/\tan \theta_2)$$

Snell's law applied at the interface gives

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

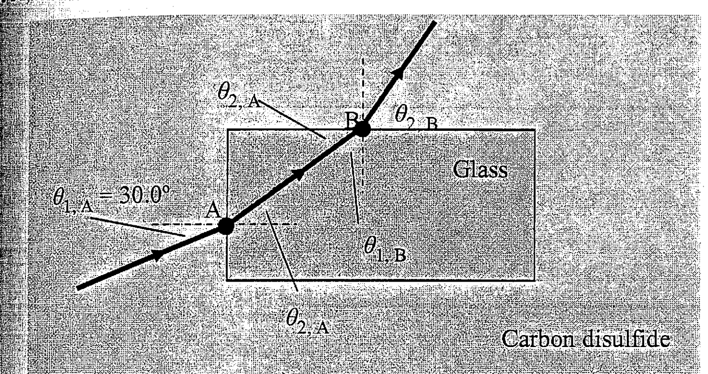
For small angles, $\sin \theta_1 \approx \tan \theta_1$ and $\sin \theta_2 \approx \tan \theta_2$, so

$$\tan \theta_1/\tan \theta_2 \approx \sin \theta_1/\sin \theta_2 = (n_2/n_1)$$

Now $d' = d\,(\tan \theta_1/\tan \theta_2)$. Therefore,

$$\boxed{d' \approx d\left(\frac{n_2}{n_1}\right)}$$

...SONING Snell's law will allow us to calculate the angle of refraction $\theta_{2,\text{B}}$ with which ... leaves the glass at point B, provided that we have a value for the angle of incidence ... this point (see the drawing). This angle of incidence is not given, but we can obtain it ...sidering what happens to the incident ray at point A. This ray is incident at an angle ...nd refracted at an angle $\theta_{2,\text{A}}$. Snell's law can be used to obtain $\theta_{2,\text{A}}$, the value for ...can be combined with the geometry at points A and B to provide the needed value for ... Since the light ray travels from a material (carbon disulfide) with a higher refractive ... toward a material (glass) with a lower refractive index, it is bent away from the normal ...nt A, as the drawing shows.



...UTION Using Snell's law at point B, we have

$$\underbrace{(1.52)\sin\theta_{1,\text{B}}}_{\text{Glass}} = \underbrace{(1.63)\sin\theta_{2,\text{B}}}_{\text{Carbon disulfide}} \quad \text{or} \quad \sin\theta_{2,\text{B}} = \left(\frac{1.52}{1.63}\right)\sin\theta_{1,\text{B}} \tag{1}$$

...find $\theta_{1,\text{B}}$ we note from the drawing that

$$\theta_{1,\text{B}} + \theta_{2,\text{A}} = 90.0° \quad \text{or} \quad \theta_{1,\text{B}} = 90.0° - \theta_{2,\text{A}} \tag{2}$$

...can find $\theta_{2,\text{A}}$, which is the angle of refraction at point A, by again using Snell's law:

$$\underbrace{(1.63)\sin\theta_{1,\text{A}}}_{\text{Carbon disulfide}} = \underbrace{(1.52)\sin\theta_{2,\text{A}}}_{\text{Glass}} \quad \text{or} \quad \sin\theta_{2,\text{A}} = \left(\frac{1.63}{1.52}\right)\sin\theta_{1,\text{A}}$$

Thus, we have

$$\sin\theta_{2,\,\mathrm{A}} = \left(\frac{1.63}{1.52}\right)\sin\theta_{1,\,\mathrm{A}} = \left(\frac{1.63}{1.52}\right)\sin 30.0° = 0.536 \quad \text{or} \quad \theta_{2,\,\mathrm{A}} = \sin^{-1}(0.536) = 32.4°$$

Using Equation (2), we find that

$$\theta_{1,\,\mathrm{B}} = 90.0° - \theta_{2,\,\mathrm{A}} = 90.0° - 32.4° = 57.6°$$

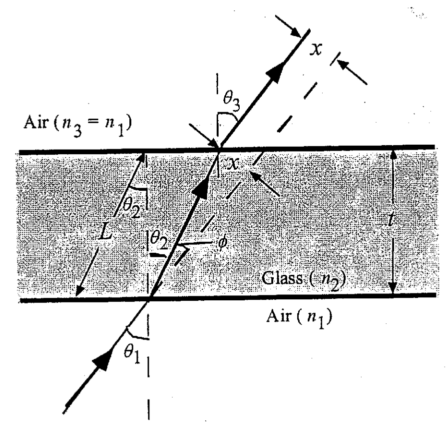
With this value for $\theta_{1,\,\mathrm{B}}$ in Equation (1) we obtain

$$\sin\theta_{2,\,\mathrm{B}} = \left(\frac{1.52}{1.63}\right)\sin\theta_{1,\,\mathrm{B}} = \left(\frac{1.52}{1.63}\right)\sin 57.6° = 0.787 \quad \text{or} \quad \theta_{2,\,\mathrm{B}} = \sin^{-1}(0.787) = \boxed{51.9°}$$

---

21. SSM WWW The drawing at the right shows the geometry of the situation using the same notation as that in Figure 26.6. In addition to the text's notation, let t represent the thickness of the pane, let L represent the length of the ray in the pane, let x (shown twice in the figure) equal the displacement of the ray, and let the difference in angles $\theta_1 - \theta_2$ be given by ϕ.

We wish to find the amount x by which the emergent ray is displaced relative to the incident ray. This can be done by applying Snell's law at each interface, and then making use of the geometric and trigonometric relations in the drawing.



SOLUTION If we apply Snell's law (see Equation 26.2) to the bottom interface we obtain $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Similarly, if we apply Snell's law at the top interface where the ray emerges, we have $n_2 \sin\theta_2 = n_3 \sin\theta_3 = n_1 \sin\theta_3$. Comparing this with Snell's law at the bottom face, we see that $n_1 \sin\theta_1 = n_1 \sin\theta_3$, from which we can conclude that $\theta_3 = \theta_1$. Therefore, the emerging ray is parallel to the incident ray.

From the geometry of the ray and thickness of the pane, we see that $L\cos\theta_2 = t$, from which it follows that $L = t/\cos\theta_2$. Furthermore, we see that $x = L\sin\phi = L\sin(\theta_1 - \theta_2)$. Substituting for L, we find

$$x = L\,\sin(\theta_1 - \theta_2) = \frac{t\,\sin(\theta_1 - \theta_2)}{\cos\theta_2}$$

... we can use this expression to determine a numerical value for x, we must find the ... of θ_2. Solving the expression for Snell's law at the bottom interface for θ_2, we have

$$\ldots = \frac{n_1\sin\theta_1}{n_2} = \frac{(1.000)(\sin 30.0°)}{1.52} = 0.329 \quad \text{or} \quad \theta_2 = \sin^{-1} 0.329 = 19.2°$$

...fore, the amount by which the emergent ray is displaced relative to the incident ray is

$$x = \frac{t\,\sin(\theta_1 - \theta_2)}{\cos\theta_2} = \frac{(6.00\text{ mm})\sin(30.0° - 19.2°)}{\cos 19.2°} = \boxed{1.19\text{ mm}}$$

...*SONING* The fish appears to be a distance less than $\frac{1}{2}L$ from the front wall of the ...rium, where L is the distance between the back and front walls. The phenomenon of *...rent depth* is at play here. According to Equation 26.3, the apparent depth or distance d' ...ated to the actual depth $\frac{1}{2}L$ by $d' = \frac{1}{2}L\left(\frac{n_{\mathrm{air}}}{n_{\mathrm{water}}}\right)$, where n_{air} and n_{water} are the refractive ...ices of air and water. Referring to Table 26.1, we see that $n_{\mathrm{air}} < n_{\mathrm{water}}$, so that $d' < \frac{1}{2}L$.

...ce the fish is a distance $\frac{1}{2}L$ in front of the plane mirror, the image of the fish is a distance ... behind the plane mirror. Thus, the image is a distance $\frac{3}{2}L$ from the front wall.

...e image of the fish appears to be at a distance less than $\frac{3}{2}L$ from the front wall, because of ...e phenomenon of *apparent depth*. The explanation given above applies here also, except ...at the actual depth or distance is $\frac{3}{2}L$ instead of $\frac{1}{2}L$.

...LUTION

a. Using Equation 26.3, we find that the apparent distance d' between the fish and the front walls is

$$d' = \tfrac{1}{2}L\left(\frac{n_{\mathrm{air}}}{n_{\mathrm{water}}}\right) = \tfrac{1}{2}(40.0\text{ cm})\left(\frac{1.000}{1.333}\right) = \boxed{15.0\text{ cm}}$$

b. Again using Equation 26.3, we find that the apparent distance between the image of the fish and the front wall is

$$d' = \tfrac{3}{2}L\left(\frac{n_{\mathrm{air}}}{n_{\mathrm{water}}}\right) = \tfrac{3}{2}(40.0\text{ cm})\left(\frac{1}{1.333}\right) = \boxed{45.0\text{ cm}}$$

As expected, the ranking of the indices of refraction, highest-to-lowest, is n_a = 1.80, $n_c = 1.38$, $n_b = 1.16$.

---

31. ***REASONING AND SOLUTION***
a. Using Equation 26.4 and the refractive index for crown glass given in Table 26.1, we find that the critical angle for a crown glass-air interface is

$$\theta_c = \sin^{-1}\left(\frac{1.00}{1.523}\right) = 41.0°$$

The light will be totally reflected at point A since the incident angle of 60.0° is greater than θ_c. The incident angle at point B, however, is 30.0° and smaller than θ_c. Thus, the light will exit first at $\boxed{\text{point B}}$.

b. The critical angle for a crown glass-water interface is

$$\theta_c = \sin^{-1}\left(\frac{1.333}{1.523}\right) = 61.1°$$

The incident angle at point A is less than this, so the light will first exit at $\boxed{\text{point A}}$.

---

32. ***REASONING*** Total internal reflection can occur only when light is traveling from a higher index material toward a lower index material. Thus, total internal reflection is possible when the material above or below a layer has a smaller index of refraction than the layer itself. With this criterion in mind, the following table indicates the various possibilities:

Layer	Is total internal reflection possible? Top surface of layer	Bottom surface of layer
a	Yes	No
b	Yes	Yes
c	No	Yes

SOLUTION The critical angle for each interface at which total internal reflection is possible is obtained from Equation 26.4:

Layer a, top surface

$$\theta_c = \sin^{-1}\left(\frac{n_{air}}{n_a}\right) = \sin^{-1}\left(\frac{1.00}{1.30}\right) = 50.3°$$

$$\Delta\theta = 75.0° - 50.3° = \boxed{24.7°}$$

Layer b, top surface

$$\theta_c = \sin^{-1}\left(\frac{n_a}{n_b}\right) = \sin^{-1}\left(\frac{1.30}{1.50}\right) = 60.1°$$

$$\Delta\theta = 75.0° - 60.1° = \boxed{14.9°}$$

Layer b, bottom surface

$$\theta_c = \sin^{-1}\left(\frac{n_c}{n_b}\right) = \sin^{-1}\left(\frac{1.40}{1.50}\right) = 69.0°$$

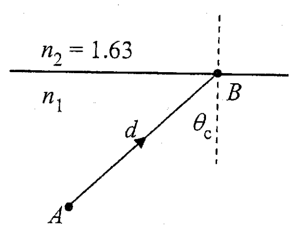
$$\Delta\theta = 75.0° - 69.0° = \boxed{6.0°}$$

Layer c, bottom surface

$$\theta_c = \sin^{-1}\left(\frac{n_{air}}{n_c}\right) = \sin^{-1}\left(\frac{1.00}{1.40}\right) = 45.6°$$

$$\Delta\theta = 75.0° - 45.6° = \boxed{29.4°}$$

REASONING The time it takes for the light to travel from *A* to *B* is equal to the distance divided by the speed of light in the substance. The distance is known, and the speed of light v in the substance is equal to the speed of light c in a vacuum divided by the index of refraction n_1 (Equation 26.1). The index of refraction can be obtained by noting that the light is incident at the critical angle θ_c (which is known). According to Equation 26.4, the index of refraction n_1 is related to the critical angle and the index of refraction n_2 by $n_1 = n_2 / \sin\theta_c$.



SOLUTION The time t it takes for the light to travel from *A* to *B* is

$$t = \frac{\text{Distance}}{\text{Speed of light in the substance}} = \frac{d}{v} \quad (1)$$

The speed of light v in the substance is related to the speed of light c in a vacuum and the index of refraction n_1 of the substance by $v = c/n_1$ (Equation 26.1). Substituting this expression into Equation (1) gives

$$t = \frac{d}{v} = \frac{d}{\left(\frac{c}{n_1}\right)} = \frac{d\,n_1}{c} \qquad (2)$$

Since the light is incident at the critical angle θ_c, we know that $n_1 \sin\theta_c = n_2$ (Equation 26.4). Solving this expression for n_1 and substituting the result into Equation (2) yields

$$t = \frac{d\,n_1}{c} = \frac{d\left(\frac{n_2}{\sin\theta_c}\right)}{c} = \frac{(4.60\text{ m})\left(\frac{1.63}{\sin 48.1°}\right)}{3.00\times10^8\text{ m/s}} = \boxed{3.36\times10^{-8}\text{ s}}$$

---

34. ***REASONING*** In the ratio n_B/n_C each refractive index can be related to a critical angle for total internal reflection according to Equation 26.4. By applying this expression to the A-B interface and again to the A-C interface, we will obtain expressions for n_B and n_C in terms of the given critical angles. By substituting these expressions into the ratio, we will be able to obtain a result from which the ratio can be calculated

SOLUTION Applying Equation 26.4 to the A-B interface, we obtain

$$\sin\theta_{c,\,AB} = \frac{n_B}{n_A} \quad \text{or} \quad n_B = n_A \sin\theta_{c,\,AB}$$

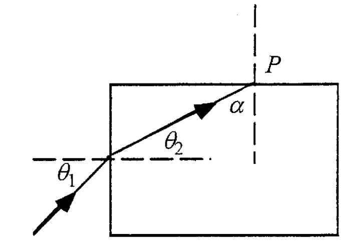
Applying Equation 26.4 to the A-C interface gives

$$\sin\theta_{c,\,AC} = \frac{n_C}{n_A} \quad \text{or} \quad n_C = n_A \sin\theta_{c,\,AC}$$

With these two results, the desired ratio can now be calculated:

$$\frac{n_B}{n_C} = \frac{n_A \sin\theta_{c,\,AB}}{n_A \sin\theta_{c,\,AC}} = \frac{\sin 36.5°}{\sin 47.0°} = \boxed{0.813}$$

SM ***REASONING*** Total internal reflection will occur at point P provided the angle α in the drawing at the right exceeds the critical angle. This angle is determined by the angle θ_2 at which the light rays enter the quartz slab. We can determine θ_2 by using Snell's law of refraction and the incident angle, which is given as $\theta_1 = 34°$.



SOLUTION Using n for the refractive index of the fluid that surrounds the crystalline quartz slab and n_q for the refractive index of quartz and applying Snell's law give

$$n\sin\theta_1 = n_q \sin\theta_2 \quad \text{or} \quad \sin\theta_2 = \frac{n}{n_q}\sin\theta_1 \qquad (1)$$

But when α equals the critical angle, we have from Equation 26.4 that

$$\sin\alpha = \sin\theta_c = \frac{n}{n_q} \qquad (2)$$

According to the geometry in the drawing above, $\alpha = 90° - \theta_2$. As a result, Equation (2) becomes

$$\sin\left(90° - \theta_2\right) = \cos\theta_2 = \frac{n}{n_q} \qquad (3)$$

Squaring Equation (3), using the fact that $\sin^2\theta_2 + \cos^2\theta_2 = 1$, and substituting from Equation (1), we obtain

$$\cos^2\theta_2 = 1 - \sin^2\theta_2 = 1 - \frac{n^2}{n_q^2}\sin^2\theta_1 = \frac{n^2}{n_q^2} \qquad (4)$$

Solving Equation (4) for n and using the value given in Table 26.1 for the refractive index of crystalline quartz, we find

$$n = \frac{n_q}{\sqrt{1+\sin^2\theta_1}} = \frac{1.544}{\sqrt{1+\sin^2 34°}} = \boxed{1.35}$$

SOLUTION When you look through the correct end of the telescope, the angular magnification of the telescope is $M_c = -f_o / f_e$. If you look through the wrong end, the roles of the objective and eyepiece lenses are interchanged, so that the angular magnification would be $M_w = -f_e / f_o$. Therefore,

$$\frac{M_c}{M_w} = \frac{-f_o/f_e}{-f_e/f_o} = \left(\frac{f_o}{f_e}\right)^2 = 32\ 800 \qquad \text{or} \qquad \frac{f_o}{f_e} = \pm\sqrt{32\ 800} = \pm 181$$

The angular magnification of the telescope is negative, so we choose the positive root and obtain $M = -f_o / f_e = -(+181) = \boxed{-181}$.

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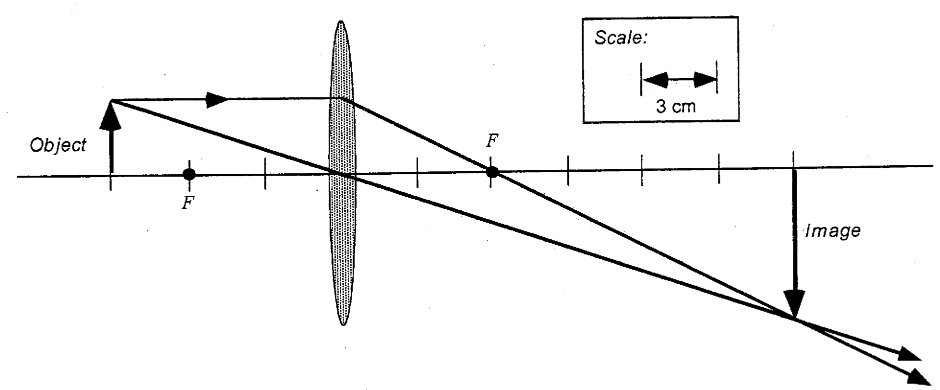
105. SSM ***REASONING*** The ray diagram is constructed by drawing the paths of two rays from a point on the object. For convenience, we choose the top of the object. The ray that is parallel to the principal axis will be refracted by the lens and pass through the focal point on the right side. The ray that passes through the center of the lens passes through undeflected. The image is formed at the intersection of these two rays on the right side of the lens.

SOLUTION The following ray diagram (to scale) shows that $\boxed{d_i = 18 \text{ cm}}$ and reveals a real, inverted, and enlarged image.



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106. ***REASONING*** The focal length f_e of the eyepiece can be determined by using Equation 26.11:

$$M \approx -\frac{(L-f_e)N}{f_o f_e} \qquad (26.11)$$

M is the angular magnification, L is the distance between the objective and the ...ece, N is the distance between the eye and the near point, and f_o is the focal length of the ...tive. We can calculate f_e from this equation, provided that we have a value for M, since ...es for all of the other variables are given in the problem statement. Although M is not ...directly, we do have a value for the angular size of the image $(\theta' = -8.8\times10^{-3} \text{ rad})$ and ...ference angular size seen by the naked eye when the object is located at the near point ...5.2×10^{-5} rad). From these two angular sizes we can determine the angular ...ification using the definition in Equation 26.10:

$$M = \frac{\theta'}{\theta} \qquad (26.10)$$

...UTION Substituting Equation 26.10 into Equation 26.11 and solving for f_e shows that

$$M = \frac{\theta'}{\theta} \approx -\frac{(L-f_e)N}{f_o f_e} \quad \text{or} \quad \theta' f_o f_e \approx -\theta(L-f_e)N \quad \text{or} \quad \theta' f_o f_e - \theta f_e N \approx -\theta L N$$

$$f_e \approx \frac{-\theta L N}{\theta' f_o - \theta N} = \frac{-(5.2\times10^{-5} \text{ rad})(16 \text{ cm})(25 \text{ cm})}{(-8.8\times10^{-3} \text{ rad})(2.6 \text{ cm}) - (5.2\times10^{-5} \text{ rad})(25 \text{ cm})} = \boxed{0.86 \text{ cm}}$$

SSM ***REASONING AND SOLUTION***

The index of refraction n_2 of the liquid must match that of the glass, or $n_2 = \boxed{1.50}$.

When none of the light is transmitted into the liquid, the angle of incidence must be equal ...or greater than the critical angle. According to Equation 26.4, the critical angle θ_c is ...en by $\sin\theta_c = n_2/n_1$, where n_2 is the index of refraction of the liquid and n_1 is that of the ...ass. Therefore,

$$n_2 = n_1 \sin\theta_c = (1.50) \sin 58.0° = \boxed{1.27}$$

...n_2 were larger than 1.27, the critical angle would also be larger, and light would be ...nsmitted from the glass into the liquid. Thus, n_2 = 1.27 represents the largest index of ...action of the liquid such that none of the light is transmitted into the liquid.
