

$$\theta = \tan^{-1} \left(\frac{y}{L} \right) = \tan^{-1} \left(\frac{0.10 \text{ m}}{0.50 \text{ m}} \right) = 11^\circ$$

$$m = \frac{d \sin \theta}{\lambda} = \frac{(1.4 \times 10^{-5} \text{ m}) \sin 11^\circ}{625 \times 10^{-9} \text{ m}} = 4.3$$

For this version, there are **4** bright fringes on the screen (on one side of the central bright fringe), again not counting the central bright fringe.

7. **REASONING AND SOLUTION** The position of the third dark fringe ($m = 2$) is given by $\sin \theta = (m + \frac{1}{2}) \frac{\lambda}{d} = (2 + \frac{1}{2}) \frac{\lambda}{d}$. The position of the fourth bright fringe ($m = 4$) is given by

$$\sin \theta = m \frac{\lambda'}{d} = 4 \frac{\lambda'}{d}. \text{ Equating these two expressions and solving for } \lambda' \text{ yields}$$

$$\lambda' = \frac{(2 + \frac{1}{2})\lambda}{4} = \frac{(2 + \frac{1}{2})(645 \text{ nm})}{4} = \boxed{403 \text{ nm}}$$

8. **REASONING AND SOLUTION** The angular position of a third order fringe is given by $\theta = \sin^{-1} 3\lambda/d$, and the position of the fringe on the screen is $y = L \tan \theta$.

For the third-order red fringe,

$$\theta = \sin^{-1} \left[\frac{3(665 \times 10^{-9} \text{ m})}{0.158 \times 10^{-3} \text{ m}} \right] = 0.724^\circ$$

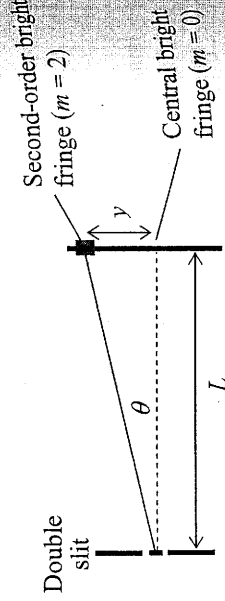
and

$$y = (2.24 \text{ m}) \tan 0.724^\circ = 2.83 \times 10^{-2} \text{ m}$$

For the third-order green fringe, $\theta = 0.615^\circ$ and $y = 2.40 \times 10^{-2} \text{ m}$.

The distance between the fringes is $2.83 \times 10^{-2} \text{ m} - 2.40 \times 10^{-2} \text{ m} = \boxed{4.3 \times 10^{-3} \text{ m}}$.

9. **REASONING** The drawing shows a top view of the double slit and the screen, as well as the position of the central bright fringe ($m = 0$) and the second-order bright fringe ($m = 2$). The vertical distance y in the drawing can be obtained from the tangent function as



$y = L \tan \theta$. According to Equation 27.1, the angle θ is related to the wavelength λ of the light and the separation d between the slits by $\sin \theta = m\lambda/d$, where $m = 0, 1, 2, 3, \dots$. If the angle θ is small, then $\tan \theta \approx \sin \theta$, so that

$$y = L \tan \theta \approx L \sin \theta = L \left(\frac{m\lambda}{d} \right) \quad (1)$$

We will use this relation to find the value of y when $\lambda = 585 \text{ nm}$.

SOLUTION When $\lambda = 425 \text{ nm}$, we know that $y = 0.0180 \text{ m}$, so

$$0.0180 \text{ m} = L \left[\frac{m(425 \text{ nm})}{d} \right] \quad (2)$$

Dividing Equation (1) by Equation (2) and algebraically eliminating the common factors of L , m , and d , we find that

$$\frac{y}{0.0180 \text{ m}} = \frac{L \left(\frac{m\lambda}{d} \right)}{L \left[\frac{m(425 \text{ nm})}{d} \right]} = \frac{\lambda}{425 \text{ nm}}$$

When $\lambda = 585 \text{ nm}$, the distance to the second-order bright fringe is

$$y = (0.0180 \text{ m}) \left(\frac{585 \text{ nm}}{425 \text{ nm}} \right) = \boxed{0.0248 \text{ m}}$$

10. **REASONING AND SOLUTION** The first-order orange fringes occur farther out from the center than do the first-order blue fringes. Therefore, the screen must be moved toward the slits so that the orange fringes will appear on the screen. The distance between the screen and the slits is L , and the amount by which the screen must be moved toward the slits is $L_{\text{blue}} - L_{\text{orange}}$. We know that $L_{\text{blue}} = 0.500 \text{ m}$, and must, therefore, determine L_{orange} . We begin with Equation 27.1, as it applies to first-order fringes, that is, $\sin \theta = \lambda/d$. Furthermore, as discussed in Example 1 in the text, $\tan \theta = y/L$, where y is the distance from the center of the screen to a fringe. Since the angle θ locating the fringes is small, $\sin \theta \approx \tan \theta$, and we have that

$$\frac{\lambda}{d} \approx \frac{y}{L} \quad \text{or} \quad L = \frac{yd}{\lambda}$$

Writing this result for L for both colors and dividing the two equations gives

$$\theta = \sin^{-1} \left(\frac{\lambda}{W} \right) = \sin^{-1} \left(\frac{v}{fW} \right) = \sin^{-1} \left[\frac{343 \text{ m/s}}{(2.0 \times 10^4 \text{ Hz})(0.91 \text{ m})} \right] = \boxed{1.1^\circ}$$

- b. For the light waves, Equation 27.4 (with $m = 1$) reveals that

$$\sin \theta = \frac{\lambda}{W} \quad \text{or} \quad W = \frac{\lambda}{\sin \theta} = \frac{580 \times 10^{-9} \text{ m}}{\sin 1.1^\circ} = \boxed{3.0 \times 10^{-5} \text{ m}}$$

Thus, in order for the diffraction of the light waves to match that of the sound waves, the light waves must pass through a very narrow “doorway.” In passing through a doorway that is 0.91 m wide, the light waves would be diffracted by an amount so small that it would be unobservable.

24. **REASONING AND SOLUTION** We have $\theta = \sin^{-1} (2\lambda/W)$ for the second dark fringe.

- a. Thus,

$$\theta = \sin^{-1} \left[\frac{2(430 \times 10^{-9} \text{ m})}{2.1 \times 10^{-6} \text{ m}} \right] = \boxed{24^\circ}$$

- b. and

$$\theta = \sin^{-1} \left[\frac{2(660 \times 10^{-9} \text{ m})}{2.1 \times 10^{-6} \text{ m}} \right] = \boxed{39^\circ}$$

25. **SSM REASONING** This problem can be solved by using Equation 27.4 for the value of the angle θ when $m = 1$ (first dark fringe).

SOLUTION

- a. When the slit width is $W = 1.8 \times 10^{-4} \text{ m}$ and $\lambda = 675 \text{ nm} = 675 \times 10^{-9} \text{ m}$, we find, according to Equation 27.4,

$$\theta = \sin^{-1} \left(m \frac{\lambda}{W} \right) = \sin^{-1} \left[(1) \frac{675 \times 10^{-9} \text{ m}}{1.8 \times 10^{-4} \text{ m}} \right] = \boxed{0.21^\circ}$$

- b. Similarly, when the slit width is $W = 1.8 \times 10^{-6} \text{ m}$ and $\lambda = 675 \times 10^{-9} \text{ m}$, we find

$$\theta = \sin^{-1} \left[(1) \frac{675 \times 10^{-9} \text{ m}}{1.8 \times 10^{-6} \text{ m}} \right] = \boxed{22^\circ}$$

REASONING Since the width of the central bright fringe on the screen is defined by the locations of the dark fringes on either side, we consider Equation 27.4, which specifies the angle θ that determines the location of a dark fringe. This equation is $\sin \theta = m\lambda/W$, where λ is the wavelength, W is the slit width, and $m = 1, 2, 3, \dots$

The fact that the width of the central bright fringe does not change means that the positions of the dark fringes to either side also do not change. In other words, the angle θ remains constant. According to Equation 27.4, the condition that must be satisfied for this to happen is that the ratio λ/W of the wavelength to the slit width remains constant.

SOLUTION Since the width of the central bright fringe on the screen remains constant, the angular position θ of the dark fringes must also remain constant. Thus, according to Equation 27.4, we have

$$\underbrace{\sin \theta = \frac{m\lambda_1}{W_1}}_{\text{Case 1}} \quad \text{and} \quad \underbrace{\sin \theta = \frac{m\lambda_2}{W_2}}_{\text{Case 2}}$$

Since θ is the same for each case, it follows that

$$\frac{m\lambda_1}{W_1} = \frac{m\lambda_2}{W_2}$$

The term m can be eliminated algebraically from this result, so that solving for W_2 gives

$$W_2 = \frac{W_1 \lambda_2}{\lambda_1} = \frac{(2.3 \times 10^{-6} \text{ m})(740 \text{ nm})}{510 \text{ nm}} = \boxed{3.3 \times 10^{-6} \text{ m}}$$

SSM WWW REASONING The width W of the slit and the wavelength λ of the light are related to the angle θ defining the location of a dark fringe in the single-slit diffraction pattern, so we can determine the width from values for the wavelength and the angle. The wavelength is given. We can obtain the angle from the width given for the central bright fringe on the screen and the distance between the screen and the slit. To do this, we will use trigonometry and the fact that the width of the central bright fringe is defined by the first dark fringe on either side of the central bright fringe.

SOLUTION The angle that defines the location of a dark fringe in the diffraction pattern can be determined according to

$$\sin \theta = m \frac{\lambda}{W} \quad m = 1, 2, 3, \dots \quad (27.4)$$

Recognizing that we need the case for $m = 1$ (the first dark fringe on either side of the central bright fringe determines the width of the central bright fringe) and solving for W gives

$$W = \frac{\lambda}{\sin \theta}$$

Referring to Figure 27.22 in the text, we see that the width of the central bright fringe is $2y$, where trigonometry shows that

$$2y = 2L \tan \theta \quad \text{or} \quad \theta = \tan^{-1} \left(\frac{2y}{2L} \right) = \tan^{-1} \left[\frac{0.050 \text{ m}}{2(0.60 \text{ m})} \right] = 2.4^\circ$$

Substituting this value for θ into Equation (1), we find that

$$W = \frac{\lambda}{\sin \theta} = \frac{510 \times 10^{-9} \text{ m}}{\sin 2.4^\circ} = \boxed{1.2 \times 10^{-5} \text{ m}}$$

28. **REASONING** The wavelength λ of the light and the width W of the single slit together determine the angle θ of the first dark fringe, via $\sin \theta = m \frac{\lambda}{W}$ (Equation 27.4), with $m = 1$:

$$\sin \theta = \frac{\lambda}{W} \quad (1)$$

Both the angle θ and the position y of the dark fringe on the screen are measured from the middle of the central bright fringe. Therefore, the angle θ may be found from $\theta = \tan^{-1} \left(\frac{y}{L} \right)$ (Equation 1.6), where L is the distance between the slit and the screen.

SOLUTION Solving Equation (1) for λ , we obtain

$$\lambda = W \sin \theta \quad (2)$$

Substituting $\theta = \tan^{-1} \left(\frac{y}{L} \right)$ (Equation 1.6) into Equation (2) yields

$$\lambda = W \sin \left[\tan^{-1} \left(\frac{y}{L} \right) \right] = (5.6 \times 10^{-4} \text{ m}) \sin \left[\tan^{-1} \left(\frac{3.5 \times 10^{-3} \text{ m}}{4.0 \text{ m}} \right) \right] = \boxed{4.9 \times 10^{-7} \text{ m}}$$

REASONING AND SOLUTION The width of the bright fringe that is next to the central bright fringe is $y_2 - y_1$, where y_1 is the distance from the center of the central bright fringe to the first-order dark fringe, and y_2 is the analogous distance for the second-order dark fringe. To find the fringe width, we take an approach similar to that outlined in Example 6 in the text. According to Equation 27.4, the dark fringes for single slit diffraction are located by $\sin \theta = m\lambda/W$. For first-order fringes, $m = 1$, and $\sin \theta_1 = \lambda/W$. For second-order fringes, $m = 2$, and $\sin \theta_2 = 2\lambda/W$. Therefore, we find that

$$\begin{aligned} \theta_1 &= \sin^{-1} \left(\frac{\lambda}{W} \right) = \sin^{-1} \left(\frac{480 \times 10^{-9} \text{ m}}{2.0 \times 10^{-5} \text{ m}} \right) = 1.4^\circ \\ \theta_2 &= \sin^{-1} \left(\frac{2\lambda}{W} \right) = \sin^{-1} \left[\frac{2(480 \times 10^{-9} \text{ m})}{2.0 \times 10^{-5} \text{ m}} \right] = 2.8^\circ \end{aligned}$$

From Example 6 we know that the distance y is given by $y = L \tan \theta$, where L is the distance between the slit and the screen. Therefore, it follows that

$$y_2 - y_1 = L(\tan \theta_2 - \tan \theta_1) = (0.50 \text{ m})(\tan 2.8^\circ - \tan 1.4^\circ) = \boxed{0.012 \text{ m}}$$

30. **REASONING** The distance y between the center of the diffraction pattern and a particular dark fringe depends upon the distance L between the slit and the screen and the angle θ at which the dark fringe appears

$$y = L \tan \theta \quad (1)$$

For light with a wavelength λ passing through a slit of width W , the angles of the dark fringes are given by $\sin \theta = m \frac{\lambda}{W}$ (Equation 27.4), where m is an integer ($m = 1, 2, 3, \dots$).

The screen will only be completely dark when *both* diffraction patterns have a dark fringe at the same angle θ . We will use Equation 27.4 to determine the integers (m_1 and m_2) for which the two wavelengths ($\lambda_1 = 632 \text{ nm}$ and $\lambda_2 = 474 \text{ nm}$) have dark fringes at the same angle. Then, Equation (1) will determine the distance y .

SOLUTION For an angle θ at which both diffraction patterns have dark fringes, Equation 27.4 gives

$$\sin \theta = m_1 \frac{\lambda_1}{W} \quad \text{and} \quad \sin \theta = m_2 \frac{\lambda_2}{W} \quad (2)$$

Setting the right sides of Equations (2) equal, we see that

$$m_1 \frac{\lambda_1}{W} = m_2 \frac{\lambda_2}{W} \quad \text{or} \quad \frac{m_1}{m_2} = \frac{\lambda_2}{\lambda_1} = \frac{474 \text{ nm}}{632 \text{ nm}} = 0.75$$

The first dark fringes of the two diffraction patterns do not coincide, because $m_1 = m_2 = 1$ yields a ratio of $m_1/m_2 = 1/1 = 1$, which does not satisfy Equation (3). But we can see that other dark fringes do coincide, because Equation (3) is satisfied when $m_1 = 3$ and $m_2 = 4$ ($m_1/m_2 = 3/4 = 0.75$), or when $m_1 = 6$ and $m_2 = 8$ ($m_1/m_2 = 6/8 = 0.75$), and so forth. The first time the dark fringes overlap occurs when $m_1 = 3$ and $m_2 = 4$. Solving the first of Equations (2) for θ , and taking $m_1 = 3$ yields

$$\theta = \sin^{-1} \left(m_1 \frac{\lambda_1}{W} \right) = \sin^{-1} \left[3 \left(\frac{632 \times 10^{-9} \text{ m}}{7.15 \times 10^{-5} \text{ m}} \right) \right] = 1.52^\circ$$

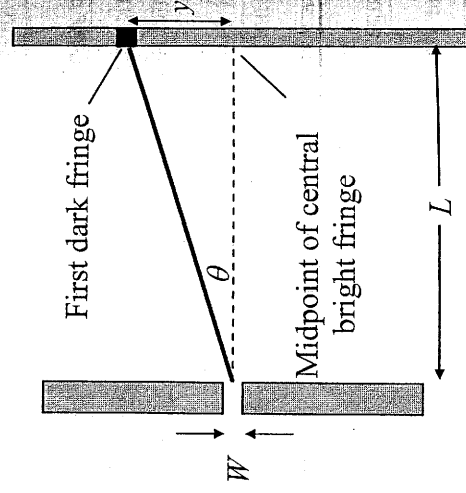
Then, from Equation (1), we have that

$$y = L \tan \theta = (1.20 \text{ m}) \tan 1.52^\circ = 3.18 \times 10^{-2} \text{ m} = \boxed{3.18 \text{ cm}}$$

31. **SSM REASONING** The width of the central bright fringe is defined by the location of the first dark fringe on either side of it. According to Equation 27.4, the angle θ locating the first dark fringe can be obtained from $\sin \theta = \lambda/W$, where λ is the wavelength of the light and W is the width of the slit. According to the drawing, $\tan \theta = y/L$, where y is half the width of the central bright fringe and L is the distance between the slit and the screen.

SOLUTION Since the angle θ is small, we can use the fact that $\sin \theta \approx \tan \theta$. Since $\sin \theta = \lambda/W$ and $\tan \theta = y/L$, we have

$$\frac{\lambda}{W} = \frac{y}{L} \quad \text{or} \quad \frac{\lambda L}{W} = y$$



Applying this result to both slits gives

$$\frac{W_2}{W_1} = \frac{\lambda L / y_2}{\lambda L / y_1} = \frac{y_1}{y_2}$$

$$W_2 = W_1 \frac{y_1}{y_2} = (3.2 \times 10^{-5} \text{ m}) \frac{\frac{1}{2}(1.2 \text{ cm})}{\frac{1}{2}(1.9 \text{ cm})} = \boxed{2.0 \times 10^{-5} \text{ m}}$$

REASONING AND SOLUTION It is given that $2y = 450W$ and $L = 18\,000W$. We know $\lambda/W = \sin \theta$. Now $\sin \theta \approx \tan \theta = y/L$, so

$$\frac{\lambda}{W} = \frac{y}{L} = \frac{225W}{18\,000W} = \boxed{0.013}$$

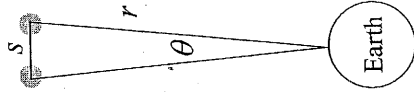
REASONING AND SOLUTION The minimum angular separation of the cars must be $\theta_{\min} = 1.22 \lambda/D$, and the separation of the cars is $y = L\theta_{\min} = 1.22 L\lambda/D$.

- a. For red light, $\lambda = 665 \text{ nm}$, and

$$y = (1.22)(8690 \text{ m})(665 \times 10^{-9} \text{ m})/(2.00 \times 10^{-3} \text{ m}) = \boxed{3.53 \text{ m}}$$

- b. For violet light, $\lambda = 405 \text{ nm}$, and

$$y = (1.22)(8690 \text{ m})(405 \times 10^{-9} \text{ m})/(2.00 \times 10^{-3} \text{ m}) = \boxed{2.15 \text{ m}}$$



REASONING The drawing shows the two stars, separated by a distance s , that are a distance r from the earth. The angle θ (in radians) is defined as the arc length divided by the radius r (see Equation 8.1). Assuming that $r \gg s$, the arc length is very nearly equal to the distance s , so we can write $\theta = s/r$. Since s is known, this relation will allow us to find the distance r if we can determine the angle. If the stars can just be seen as separate objects, we know that the minimum angle $\theta_{\min}$ between them is given by Equation 27.6 as $\theta_{\min} \approx 1.22\lambda/D$, where λ is the wavelength of the light being observed and D is the diameter of the telescope's objective. By combining these two relations, we will be able to find the distance r .

two rays of light shining on it. At nearly perpendicular incidence, ray 2 travels a distance $2t$ farther than ray 1, where t is the thickness of the film. In addition, ray 2 experiences a phase shift of $\frac{1}{2}\lambda_{\text{film}}$ upon reflection at the bottom film surface, while ray 1 experiences the same phase shift at the upper film surface. This is because, in both cases, the light is traveling through a region where the refractive index is lower toward a region where it is higher. Therefore, there is no net phase change for the two reflected rays, and only the travel distance determines the type of interference that occurs. For constructive interference, the extra travel distance must be an integer number of wavelengths in the film:

$$\underbrace{2t}_{\text{Extra distance traveled by ray 2}} + \underbrace{0}_{\text{Zero net phase change due to reflection}} = \underbrace{\lambda_{\text{film}}, 2\lambda_{\text{film}}, 3\lambda_{\text{film}}, \dots}_{\text{Condition for constructive interference}}$$

This result is equivalent to

$$2t = m\lambda_{\text{film}} \quad m = 1, 2, 3, \dots$$

The wavelength in the film, then, is

$$\lambda_{\text{film}} = \frac{2t}{m} \quad m = 1, 2, 3, \dots \quad (2)$$

SOLUTION Substituting Equation (2) into Equation (1) gives

$$n_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{\lambda_{\text{film}}} = \frac{\lambda_{\text{vacuum}}}{\frac{2t}{m}}$$

Since the given value for t is the minimum thickness for which constructive interference can occur, we know that $m = 1$. Thus, we find that

$$n_{\text{film}} = \frac{m\lambda_{\text{vacuum}}}{2t} = \frac{(1)(625 \text{ nm})}{2(242 \text{ nm})} = \boxed{1.29}$$

55. **SSM REASONING** The angles θ that determine the locations of the dark and bright fringes in a Young's double-slit experiment are related to the integers m that identify the fringes, the wavelength λ of the light, and the separation d between the slits. Since values are given for m , λ , and d , the angles can be calculated.

SOLUTION The expressions that specify θ in terms of m , λ , and d are as follows:

$$\sin \theta = m \frac{\lambda}{d} \quad m = 0, 1, 2, 3, \dots \quad (27.1)$$

Bright fringes

$$\sin \theta = \left(m + \frac{1}{2}\right) \frac{\lambda}{d} \quad m = 0, 1, 2, 3, \dots \quad (27.2)$$

Dark fringes

Applying these expressions gives the answers that we seek.

$$\sin \theta = \left(m + \frac{1}{2}\right) \frac{\lambda}{d} \quad \text{or} \quad \theta = \sin^{-1} \left[\left(0 + \frac{1}{2}\right) \frac{520 \times 10^{-9} \text{ m}}{1.4 \times 10^{-6} \text{ m}} \right] = \boxed{11^\circ}$$

a.

$$\sin \theta = m \frac{\lambda}{d} \quad \text{or} \quad \theta = \sin^{-1} \left[(1) \frac{520 \times 10^{-9} \text{ m}}{1.4 \times 10^{-6} \text{ m}} \right] = \boxed{22^\circ}$$

b.

$$\sin \theta = \left(m + \frac{1}{2}\right) \frac{\lambda}{d} \quad \text{or} \quad \theta = \sin^{-1} \left[\left(1 + \frac{1}{2}\right) \frac{520 \times 10^{-9} \text{ m}}{1.4 \times 10^{-6} \text{ m}} \right] = \boxed{34^\circ}$$

c.

$$\sin \theta = m \frac{\lambda}{d} \quad \text{or} \quad \theta = \sin^{-1} \left[(2) \frac{520 \times 10^{-9} \text{ m}}{1.4 \times 10^{-6} \text{ m}} \right] = \boxed{48^\circ}$$

d.

56. **REASONING** In a Young's double-slit experiment a dark fringe is located at an angle θ that is determined according to

$$\sin \theta = \left(m + \frac{1}{2}\right) \frac{\lambda}{d} \quad m = 0, 1, 2, 3, \dots \quad (27.2)$$

where λ is the wavelength of the light and d is the separation of the slits. Here, neither λ nor d is known. However, we know the angle for the dark fringe for which $m = 0$. Using this angle and $m = 0$ in Equation 27.2 will allow us to determine the ratio λ/d , which can then be used to find the angle that locates the dark fringe for $m = 1$.

SOLUTION Applying Equation 27.2 to the dark fringes for $m = 0$ and $m = 1$, we have

$$\sin \theta_0 = \left(0 + \frac{1}{2}\right) \frac{\lambda}{d} \quad \text{and} \quad \sin \theta_1 = \left(1 + \frac{1}{2}\right) \frac{\lambda}{d}$$

Dividing the expression for $\sin \theta_1$ by the expression for $\sin \theta_0$ gives