

13. (b) The mass m of an object is proportional to the object's rest energy E_0 , according to $E_0 = mc^2$. The rest energy includes all forms of energy except kinetic energy, which plays no role here, because the glass is not moving. To freeze half the liquid into ice, energy in the form of heat must be removed from the liquid, so the water in possibility B has more mass than the water in possibility A. To freeze the remaining liquid into ice, more heat must be removed, so the water in possibility A has more mass than the water in possibility C. Thus, the ranking in descending order (largest first) is B, A, C.

14. $1.88 \times 10^{-5} \text{ kg}$

15. (b) The total energy is the sum of the rest energy and the kinetic energy. The rest energy includes all forms of energy (including potential energy) except kinetic energy.

16. $2.60 \times 10^8 \text{ m/s}$

17. (a) According to Equation 28.7, the magnitude of the momentum is $p = \frac{\sqrt{E^2 - m^2 c^4}}{c}$, where E is the total energy. The total energy is $E = E_0 + \text{KE}$. Since the kinetic energy is equal to the rest energy, the total energy is $E = 2E_0$. Substituting this result into the

expression for p and using the fact that $mc^2 = E_0$ give $p = \frac{\sqrt{4E_0^2 - E_0^2}}{c} = \frac{\sqrt{3}mc^2}{c}$.

18. $2.18 \times 10^8 \text{ m/s}$

CHAPTER 28 | SPECIAL RELATIVITY

PROBLEMS

SSM REASONING Since the "police car" is moving relative to the earth observer, the earth observer measures a greater time interval Δt between flashes. Since both the proper time Δt_0 (as observed by the officer) and the dilated time Δt (as observed by the person on earth) are known, the speed of the "police car" relative to the observer can be determined from the time dilation relation, Equation 28.1.

SOLUTION According to Equation 28.1, the dilated time interval between flashes is $\Delta t = \Delta t_0 / \sqrt{1 - (v^2/c^2)}$, where Δt_0 is the proper time. Solving for the speed v , we find

$$v = c \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t} \right)^2} = (3.0 \times 10^8 \text{ m/s}) \sqrt{1 - \left(\frac{1.5 \text{ s}}{2.5 \text{ s}} \right)^2} = \boxed{2.4 \times 10^8 \text{ m/s}}$$

REASONING

a. The two events in this problem are the creation of the pion and its subsequent decay (or breaking apart). Imagine a reference frame attached to the pion, so the pion is stationary relative to this reference frame. To a hypothetical person who is at rest with respect to this reference frame, these two events occur at the same place, namely, at the place where the pion is located. Thus, this hypothetical person measures the proper time interval Δt_0 for the decay of the pion. On the other hand, the person standing in the laboratory sees the two events occurring at different locations, since the pion is moving relative to that person. The laboratory person, therefore, measures a dilated time interval Δt . The relation between these two time intervals is given by $\Delta t = \Delta t_0 / \sqrt{1 - v^2/c^2}$ (Equation 28.1).

b. According to the hypothetical person who is at rest in the reference frame attached to the moving pion, the distance x that the laboratory travels before the pion breaks apart is equal to the speed v of the laboratory relative to the pion times the proper time interval Δt_0 , or $x = v\Delta t_0$. The speed of the laboratory relative to the pion is the same as the speed of the pion relative to the laboratory, namely, $0.990c$.

SOLUTION

a. The proper time interval is

SOLUTION Dividing the expression for L_B by the expression for L_A and eliminating Δt , we obtain

$$\frac{L_B}{L_A} = \frac{L_0 \sqrt{1 - \frac{v_B^2}{c^2}}}{L_0 \sqrt{1 - \frac{v_A^2}{c^2}}} = \frac{\sqrt{1 - \frac{v_B^2}{c^2}}}{\sqrt{1 - \frac{v_A^2}{c^2}}}$$

$$L_B = L_A \frac{\sqrt{1 - \frac{v_B^2}{c^2}}}{\sqrt{1 - \frac{v_A^2}{c^2}}} = (6.5 \text{ light-years}) \frac{\sqrt{1 - \frac{(0.90c)^2}{c^2}}}{\sqrt{1 - \frac{(0.70c)^2}{c^2}}} = \boxed{4.0 \text{ light-years}}$$

14. REASONING

a. The two events are the creation of the particle and its subsequent disintegration. Relative to a stationary reference frame fixed to the laboratory, these two events occur at different locations, because the particle is moving relative to this reference frame. The proper distance L_0 is the distance (1.05×10^{-3} m) given in the statement of the problem, because this distance is measured by an observer in the laboratory who is at rest with respect to these locations.

b. The distance measured by a hypothetical person traveling with the particle is a contracted distance, because it is measured by a person who is moving relative to the two locations. The contracted distance L is related to the proper distance L_0 by the length-contraction formula, $L = L_0 \sqrt{1 - v^2/c^2}$ (Equation 28.2).

c. The proper lifetime Δt_0 of the particle is the lifetime as registered in a reference frame attached to the particle. In this reference frame the two events occur at the same location. The proper lifetime is equal to the contracted distance L , which is measured in this reference frame, divided by the speed v of the particle, or $\Delta t_0 = L/v$.

d. The particle's contracted lifetime Δt is related to its proper lifetime Δt_0 by the time-dilation formula, $\Delta t = \Delta t_0 / \sqrt{1 - v^2/c^2}$ (Equation 28.1).

SOLUTION

a. The proper distance is $L_0 = \boxed{1.05 \times 10^{-3} \text{ m}}$.

b. The distance measured by a hypothetical person traveling with the particle is

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = (1.05 \times 10^{-3} \text{ m}) \sqrt{1 - \frac{(0.990c)^2}{c^2}} = \boxed{1.48 \times 10^{-4} \text{ m}} \quad (28.2)$$

c. The proper lifetime Δt_0 is equal to the contracted distance L divided by the speed v of the particle:

$$\Delta t_0 = \frac{L}{v} = \frac{1.48 \times 10^{-4} \text{ m}}{(0.990)(3.00 \times 10^8 \text{ m/s})} = \boxed{4.98 \times 10^{-13} \text{ s}}$$

d. The dilated lifetime is

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{(4.98 \times 10^{-13} \text{ s})}{\sqrt{1 - \frac{(0.990c)^2}{c^2}}} = \boxed{3.53 \times 10^{-12} \text{ s}} \quad (28.1)$$

REASONING AND SOLUTION The side x will have its length contracted as viewed from the rocket. Taking the proper length to be $L_0 = x$, we can find the dilated length $L = x'$ from Equation 28.2:

$$x' = x \sqrt{1 - \left(\frac{v}{c}\right)^2} = x \sqrt{1 - \left(\frac{0.730c}{c}\right)^2} = 0.683x$$

The new angle, can be found using $\tan \theta' = y/x' = y/(0.683x) = (\tan 30.0^\circ)/(0.683) = 0.845$.

$$\theta' = \tan^{-1} 0.845 = \boxed{40.2^\circ}$$

REASONING To the observer at rest relative to the cube, its length, width, and height are all equal to the proper length $L_0 = 0.11$ m of one of the cube's sides. Suppose that the moving observer is moving parallel to the width of the cube and, therefore, measures a contracted length L for the width. Note, however, that this observer still measures the proper length L_0 for the other two dimensions of the cube, since they are perpendicular to the direction of motion. The shortened width of the cube is given by $L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$ (Equation 28.2), where v is the speed of the observer relative to the cube. Accordingly, the volume $V = L_0 \times L \times L_0 = L_0^2 L$ of the cube is smaller for the moving observer than it is for the observer at rest relative to the cube. Given the mass m of the cube, then, the moving

23. **SSM REASONING AND SOLUTION** The total momentum of the man/woman system is conserved, since no net external force acts on the system. Therefore, the final total momentum $p_m + p_w$ must equal the initial total momentum, which is zero. As a result, $p_m = -p_w$ where Equation 28.3 must be used for the momenta p_m and p_w . Thus, we find

$$\frac{m_m v_m}{\sqrt{1 - (v_m/c)^2}} = - \frac{m_w v_w}{\sqrt{1 - (v_w/c)^2}}$$

We know that $m_m = 88$ kg, $m_w = 54$ kg, and $v_w = +2.5$ m/s. Remember that c has the hypothetical value of 3.0 m/s. Solving for v_m , we find $v_m = -2.0$ m/s.

24. **REASONING AND SOLUTION** The total energy for each particle is given by

$$E = \frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2}{\sqrt{1 - \left(\frac{0.20c}{c}\right)^2}} = 8.4 \times 10^{-14} \text{ J} \quad (28.4)$$

The annihilation energy is twice this value, so $E' = 2E = 1.7 \times 10^{-13} \text{ J}$.

25. **SSM REASONING AND SOLUTION**

a. In Section 28.6 it is shown that when the speed of a particle is $0.01c$ (or less), the relativistic kinetic energy becomes nearly equal to the nonrelativistic kinetic energy. Since the speed of the particle here is $0.001c$, the ratio of the relativistic kinetic energy to the nonrelativistic kinetic energy is 1.0 .

b. Taking the ratio of the relativistic kinetic energy, Equation 28.6, to the nonrelativistic kinetic energy, $\frac{1}{2}mv^2$, we find that

$$\begin{aligned} \frac{mc^2 \left(\frac{1}{\sqrt{1 - (v^2/c^2)}} - 1 \right)}{\frac{1}{2}mv^2} &= 2 \left(\frac{c}{v} \right)^2 \left(\frac{1}{\sqrt{1 - (v^2/c^2)}} - 1 \right) \\ &= 2 \left(\frac{c}{0.970c} \right)^2 \left(\frac{1}{\sqrt{1 - (0.970c)^2/c^2}} - 1 \right) = 6.6 \end{aligned}$$

REASONING Compressing the spring stores elastic potential energy, which increases the total energy of the spring. Because the spring is not in motion, it is the spring's rest energy E_0 that increases, as well as its mass m , according to $E_0 = mc^2$ (Equation 28.5). We will use the hypothetical value of $c = 3.00 \times 10^2$ m/s for the speed of light in a vacuum in Equation 28.5. The increase of $\Delta m = 0.010$ g $= 0.010 \times 10^{-3}$ kg in the mass of the spring, therefore, corresponds to an increase ΔE_0 in the rest energy of the spring, where

$$\Delta E_0 = (\Delta m)c^2 \quad (1)$$

The increase of the rest energy is equal to the spring's elastic potential energy $PE_{\text{elastic}} = \frac{1}{2}kx^2$ (Equation 10.13), where k is the spring constant of the spring, and x is the distance by which the spring is compressed from its equilibrium length. Therefore, we have that

$$\Delta E_0 = PE_{\text{elastic}} = \frac{1}{2}kx^2 \quad (2)$$

SOLUTION Setting the right sides of Equations (1) and (2) equal to one another and solving for x^2 yields

$$\Delta E_0 = \frac{1}{2}kx^2 = (\Delta m)c^2 \quad \text{or} \quad x^2 = \frac{2(\Delta m)c^2}{k} \quad (3)$$

Taking the square root of Equation (3), we obtain

$$x = \sqrt{\frac{2(\Delta m)c^2}{k}} = c \sqrt{\frac{2(\Delta m)}{k}} = (3.00 \times 10^2 \text{ m/s}) \sqrt{\frac{2(0.010 \times 10^{-3} \text{ kg})}{850 \text{ N/m}}} = 0.046 \text{ m}$$

REASONING The mass m of the aspirin is related to its rest energy E_0 by Equation 28.5, $E_0 = mc^2$. Since it requires 1.1×10^8 J to operate the car for twenty miles, we can calculate the number of miles that the car can go on the energy that is equivalent to the mass of one tablet.

SOLUTION We begin by converting the mass m from milligrams (mg) to kilograms (kg):

$$(325 \text{ mg}) \left(\frac{1 \text{ g}}{1000 \text{ mg}} \right) \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right) = 325 \times 10^{-6} \text{ kg}$$

The number N of miles the car can go on one aspirin tablet is

$$N = \frac{E_0}{(1.1 \times 10^8 \text{ J}) / (20.0 \text{ mi})} = \frac{mc^2}{(1.1 \times 10^8 \text{ J}) / (20.0 \text{ mi})}$$

$$= \frac{(325 \times 10^{-6} \text{ kg})(3.0 \times 10^8 \text{ m/s})^2}{(1.1 \times 10^8 \text{ J}) / (20.0 \text{ mi})} = \boxed{5.3 \times 10^6 \text{ mi}}$$

28. **REASONING** In order to change a certain mass of ice at 0°C into liquid water at 0°C heat must be added, and heat is a form of energy. Therefore, the energy of the liquid water is greater than that of the ice. According to special relativity, energy and mass are equivalent. Since the liquid water has the greater energy, it also has the greater mass.

Heat must also be added to boil water into steam. Following the same type of reasoning as in the case of melting, we conclude that the steam has the greater mass.

The amount of heat Q that must be supplied to change the phase of m kilograms of a substance is given by Equation 12.5 as $Q = mL$, where L is the latent heat of the substance. Since the latent heat of vaporization L_v for water is greater than the latent heat of fusion L_f (see Table 12.3), the change in mass is greater when liquid water turns into steam at 100°C than when ice turns into liquid water at 0°C .

SOLUTION The change in mass Δm associated with a change in rest energy ΔE_0 is given by Equation 28.5 as $\Delta m = \Delta E_0 / c^2$. The change in rest energy is the heat Q that must be added to change the phase of the water, so that $Q = mL$. Thus, the change in mass is $\Delta m = Q / c^2 = mL / c^2$.

- a. According to Table 12.3, the latent heat of fusion for water is $L_f = 3.35 \times 10^5 \text{ J/kg}$. The change in mass associated with the ice to liquid-water phase change at 0°C is, then,

$$\Delta m = \frac{mL_f}{c^2} = \frac{(2.00 \text{ kg})(3.35 \times 10^5 \text{ J/kg})}{(3.00 \times 10^8 \text{ m/s})^2} = \boxed{7.44 \times 10^{-12} \text{ kg}}$$

- b. According to Table 12.3, the latent heat of vaporization for water is $L_v = 2.26 \times 10^6 \text{ J/kg}$. The change in mass associated with the liquid-water to steam phase change at 100°C is

$$\Delta m = \frac{mL_v}{c^2} = \frac{(2.00 \text{ kg})(2.26 \times 10^6 \text{ J/kg})}{(3.00 \times 10^8 \text{ m/s})^2} = \boxed{5.02 \times 10^{-11} \text{ kg}}$$

As expected, the change in mass for the liquid-to-steam phase change is greater than that for the ice-to-liquid phase change.

29. **SSM REASONING** According to the work-energy theorem, Equation 6.3, the work that must be done on the electron to accelerate it from rest to a speed of $0.990c$ is equal to the kinetic energy of the electron when it is moving at $0.990c$.

SOLUTION Using Equation 28.6, we find that

$$\begin{aligned} \text{KE} &= mc^2 \left(\frac{1}{\sqrt{1 - (v^2/c^2)}} - 1 \right) \\ &= (9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \left(\frac{1}{\sqrt{1 - (0.990c)^2/c^2}} - 1 \right) = \boxed{5.0 \times 10^{-13} \text{ J}} \end{aligned}$$

30. **REASONING AND SOLUTION** The energy E_0 produced in one year is the product of the power P generated and the time t , $E_0 = Pt$. This energy is equivalent to an amount of mass m given by Equation 28.5 as $E_0 = mc^2$. Thus, we have that

$$mc^2 = Pt \quad \text{or} \quad m = \frac{Pt}{c^2}$$

The mass of nuclear fuel consumed in one year ($3.15 \times 10^7 \text{ s}$) is

$$m = \frac{Pt}{c^2} = \frac{(3.0 \times 10^9 \text{ W})(3.15 \times 10^7 \text{ s})}{(3.00 \times 10^8 \text{ m/s})^2} = \boxed{1.1 \text{ kg}}$$

31. **REASONING** The rate R that the quasar is losing mass is equal to the mass m it loses divided by the time t during which the loss occurs, or $R = m/t$. The mass that the quasar loses is equivalent to a certain amount of energy E_0 ; this equivalency is expressed by $m = E_0/c^2$ (Equation 28.5), where c is the speed of light. According to Equation 6.10b, the energy radiated is equal to the average power $\bar{P}$ times the time, $E_0 = \bar{P}t$. The average power is the rate at which the quasar radiates energy.

39. **SSM** **WWW** **REASONING** Since the crew is initially at rest relative to the escape pod, the length of 45 m is the proper length L_0 of the pod. The length of the escape pod as determined by an observer on earth can be obtained from the relation for length contraction given by Equation 28.2, $L = L_0 \sqrt{1 - (v_{PE}^2/c^2)}$. The quantity v_{PE} is the speed of the escape pod relative to the earth, which can be found from the velocity-addition formula, Equation 28.8. The following are the relative velocities, assuming that the direction away from the earth is the positive direction:

v_{PE} = velocity of the escape Pod relative to Earth.

v_{PR} = velocity of escape Pod relative to the Rocket = $-0.55c$. This velocity is negative because the rocket is moving away from the earth (in the positive direction), and the escape pod is moving in an opposite direction (the negative direction) relative to the rocket.

v_{RE} = velocity of Rocket relative to Earth = $+0.75c$

These velocities are related by the velocity-addition formula, Equation 28.8.

SOLUTION The relative velocity of the escape pod relative to the earth is

$$v_{PE} = \frac{v_{PR} + v_{RE}}{1 + \frac{v_{PR}v_{RE}}{c^2}} = \frac{-0.55c + 0.75c}{1 + \frac{(-0.55c)(+0.75c)}{c^2}} = +0.34c$$

The speed of the pod relative to the earth is the magnitude of this result, or $0.34c$. The length of the pod as determined by an observer on earth is

$$L = L_0 \sqrt{1 - \frac{v_{PE}^2}{c^2}} = (45 \text{ m}) \sqrt{1 - \frac{(0.34c)^2}{c^2}} = \boxed{42 \text{ m}}$$

40. **REASONING** The passengers would measure a length for the spaceships that does not match the constructed length if the two ships have a nonzero relative speed. This would mean that the ships are moving with respect to one another. If so, the phenomenon of length contraction would occur, and the passengers in either ship would measure a contracted length.

To calculate the contracted length L , the length contraction formula must be used, as given in Equation 28.2:

$$L = L_0 \sqrt{1 - \frac{v_{AB}^2}{c^2}} \quad (28.2)$$

where L_0 is the proper length of the spaceships, that is, the length measured by an observer at rest with respect to them. In other words, the proper length is the constructed length. The velocity v_{AB} in Equation 28.2 is the velocity of spaceship A with respect to spaceship B. It can be obtained from the velocities of each ship with respect to the earth by using the velocity addition equation, as given in Equation 28.8:

$$v_{AB} = \frac{v_{AE} + v_{EB}}{1 + \frac{v_{AE}v_{EB}}{c^2}} \quad (28.8)$$

where v_{AE} is the velocity of spaceship A with respect to the earth and v_{EB} is the velocity of the earth with respect to spaceship B. We note that $v_{EB} = -v_{BE}$, where v_{BE} is the velocity of spaceship B with respect to the earth. Substituting $v_{EB} = -v_{BE}$ into Equation 28.8 gives

$$v_{AB} = \frac{v_{AE} - v_{BE}}{1 - \frac{v_{AE}v_{BE}}{c^2}} \quad (1)$$

SOLUTION Using Equation (1) to calculate v_{AB} for use in Equation 28.2, we find

$$v_{AB} = \frac{v_{AE} - v_{BE}}{1 - \frac{v_{AE}v_{BE}}{c^2}} = \frac{0.850c - 0.500c}{1 - \frac{(0.850c)(0.500c)}{c^2}} = 0.609c$$

Here we have taken the direction in which the spaceships are traveling to be the positive direction. Substituting this result into Equation 28.2 reveals that

$$L = L_0 \sqrt{1 - \frac{v_{AB}^2}{c^2}} = (1.50 \text{ km}) \sqrt{1 - \frac{(0.609c)^2}{c^2}} = \boxed{1.19 \text{ km}}$$

41. REASONING

The following relative velocities are pertinent to this problem. It is assumed that particle 1 is moving in the positive direction, and, therefore, particle 2 is moving in the negative direction.

$v_{P_1P_2}$ = velocity of particle 1 (P_1) relative to particle 2 (P_2)

v_{P_1L} = velocity of particle 1 (P_1) relative to an observer in the Laboratory

$= +2.10 \times 10^8 \text{ m/s}$

v_{P_2L} = velocity of particle 2 (P_2) relative to an observer in the Laboratory

$= -2.10 \times 10^8 \text{ m/s}$

The velocity $v_{P_1P_2}$ can be obtained from the velocity-addition formula, Equation 28.8:

$$v_{P_1P_2} = \frac{v_{P_1L} + v_{LP_2}}{1 + \frac{v_{P_1L}v_{LP_2}}{c^2}}$$

The velocity v_{P_1L} is given, but v_{LP_2} , the velocity of the laboratory observer relative to particle 2, is not. However, we know that v_{LP_2} is the negative of v_{P_2L} , so $v_{LP_2} = -v_{P_2L} = -(-2.10 \times 10^8 \text{ m/s}) = +2.10 \times 10^8 \text{ m/s}$.

SOLUTION

a. According to the velocity-addition formula, the velocity of particle 1 relative to particle 2 is

$$v_{P_1P_2} = \frac{v_{P_1L} + v_{LP_2}}{1 + \frac{v_{P_1L}v_{LP_2}}{c^2}} = \frac{+2.10 \times 10^8 \text{ m/s} + 2.10 \times 10^8 \text{ m/s}}{1 + \frac{(+2.10 \times 10^8 \text{ m/s})(+2.10 \times 10^8 \text{ m/s})}{c^2}} = +2.82 \times 10^8 \text{ m/s}$$

The speed of one particle as seen by the other particle is the magnitude of this result, or

$$\boxed{2.82 \times 10^8 \text{ m/s}}.$$

b. The relativistic momentum is given by Equation 28.3, where the speed is that determined in part a. Therefore,

$$p = \frac{mv_{P_1P_2}}{\sqrt{1 - \left(\frac{v_{P_1P_2}}{c}\right)^2}} = \frac{(2.16 \times 10^{-25} \text{ kg})(+2.82 \times 10^8 \text{ m/s})}{\sqrt{1 - \left(\frac{2.82 \times 10^8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}\right)^2}} = \boxed{1.8 \times 10^{-16} \text{ kg} \cdot \text{m/s}}$$

42. **REASONING AND SOLUTION** The mass equivalent is given by $E_0 = KE = mc^2$ or

$$m = \frac{KE}{c^2} = \frac{7.8 \times 10^{-13} \text{ J}}{(3.00 \times 10^8 \text{ m/s})^2} = \boxed{8.7 \times 10^{-30} \text{ kg}} \quad (28.5)$$

43. **SSM WWW REASONING AND SOLUTION** The diameter D of the planet, as measured by a moving spacecraft, is given in terms of the proper diameter D_0 by Equation 28.2. Taking the ratio of the diameter D_A of the planet measured by spaceship A to the diameter D_B measured by spaceship B, we find

$$\frac{D_A}{D_B} = \frac{D_0 \sqrt{1 - \frac{v_A^2}{c^2}}}{D_0 \sqrt{1 - \frac{v_B^2}{c^2}}} = \frac{\sqrt{1 - \frac{(0.60c)^2}{c^2}}}{\sqrt{1 - \frac{(0.80c)^2}{c^2}}} = \boxed{1.3}$$

44. **REASONING** The magnitude of the relativistic momentum p of the proton is related to its relativistic total energy E by $E^2 = p^2c^2 + m^2c^4$ (Equation 28.7), where $m = 1.67 \times 10^{-27} \text{ kg}$ is the mass of a proton (see the inside front cover of the text) and c is the speed of light in a vacuum.

SOLUTION Solving Equation 28.7 for p , we obtain

$$p^2c^2 = E^2 - m^2c^4 \quad \text{or} \quad p^2 = \frac{E^2}{c^2} - m^2c^2 \quad \text{or} \quad p = \sqrt{\frac{E^2}{c^2} - m^2c^2}$$

Therefore, the magnitude of the relativistic momentum of the proton is

$$p = \sqrt{\frac{(2.7 \times 10^{-10} \text{ J})^2}{(3.00 \times 10^8 \text{ m/s})^2} - (1.67 \times 10^{-27} \text{ kg})^2 (3.00 \times 10^8 \text{ m/s})^2} = \boxed{7.5 \times 10^{-19} \text{ kg} \cdot \text{m/s}}$$

45. **REASONING** The total time for the trip is one year. This time is the proper time interval Δt_0 , because it is measured by an observer (the astronaut) who is at rest relative to the beginning and ending events (the times when the trip started and ended) and who sees them at the same location in spacecraft. On the other hand, the astronaut measures the clocks on earth to run at the dilated time interval Δt , which is the time interval of one hundred years. The relation between the two time intervals is given by Equation 28.1, which can be used to find the speed of the spacecraft.