$$\lambda_2 = \frac{1}{\dfrac{\left(3.28\times10^{-19}\text{ J}\right)}{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}+\dfrac{1}{221\times10^{-9}\text{ m}}} = 1.62\times10^{-7}\text{ m} = \boxed{162\text{ nm}}$$

---

11. ***REASONING*** The wavelength λ of the photon is related to its frequency f by $\lambda = c/f$ (Equation 16.1), where c is the speed of light. The frequency of the photon is proportional to its energy E via $f = E/h$ (Equation 29.2), where h is Planck's constant. Thus, $\lambda = ch/E$. The photon energy is equal to the sum of the maximum kinetic energy KE_{max} of the ejected electron and the work function W_0 of the metal; $E = KE_{max} + W_0$ (Equation 29.3). Substituting this expression for E into $\lambda = ch/E$ gives

$$\lambda = \frac{c\,h}{KE_{max} + W_0} \tag{1}$$

The maximum kinetic energy is related to the maximum speed v_{max} by $KE_{max} = \frac{1}{2}mv_{max}^2$ (Equation 6.2), where m is the mass of the electron.

SOLUTION Substituting $KE_{max} = \frac{1}{2}mv_{max}^2$ into Equation (1), and converting the work function from electron-volts to joules, gives

$$\lambda = \frac{c\,h}{\frac{1}{2}mv_{max}^2 + W_0}$$

$$= \frac{\left(3.00\times10^{8}\text{ m/s}\right)\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)}{\frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(1.2\times10^{6}\text{ m/s}\right)^2 + (2.3\text{ eV})\left(\dfrac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)} = \boxed{1.9\times10^{-7}\text{ m}}$$

---

12. ***REASONING*** The total energy Q delivered by N photons is NE, where E is the energy carried by one photon, so that $N = Q/E$. Equation 29.2 indicates that the photon energy is $E = hf$, where h is Planck's constant and f is the frequency. Thus, the number N of photons can be written as

$$N = \frac{Q}{E} = \frac{Q}{h\,f} \tag{1}$$

Equation 16.1 relates the frequency f of a photon to its wavelength λ according to $f = c/\lambda$, where c is the speed of light in a vacuum. Therefore, Equation (1) can be expressed as

$$N = \frac{Q}{h\,f} = \frac{\lambda Q}{h\,c} \tag{2}$$

According to Equation 12.4, the heat Q required to raise the temperature of a substance by an amount ΔT is $Q = c_{\text{specific heat}} m\Delta T$, where $c_{\text{specific heat}}$ is the specific heat capacity of the substance and m is its mass.

SOLUTION Combining Equation 12.4 with Equation (2), the number of photons required to raise the temperature by an amount ΔT is

$$N = \frac{\lambda Q}{h\,c} \qquad \text{or} \qquad N = \frac{\lambda\, c_{\text{specific heat}}\, m\,\Delta T}{hc}$$

Applying this result to each type of photon, we obtain

$$N_{\text{infrared}} = \frac{\left(6.0\times10^{-5}\text{ m}\right)\left[840\text{ J/(kg}\cdot\text{C°)}\right](0.50\text{ kg})(2.0\text{ C°})}{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)} = \boxed{2.5\times10^{23}}$$

$$N_{\text{blue}} = \frac{\left(4.7\times10^{-7}\text{ m}\right)\left[840\text{ J/(kg}\cdot\text{C°)}\right](0.50\text{ kg})(2.0\text{ C°})}{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)} = \boxed{2.0\times10^{21}}$$

---

13. SSM WWW ***REASONING AND SOLUTION***
a. According to Equation 24.5b, the electric field can be found from $E = \sqrt{S/(\varepsilon_0 c)}$. The intensity S of the beam is

$$S = \frac{\text{Energy per unit time}}{A} = \frac{Nhf}{A} = \frac{Nh}{A}\left(\frac{c}{\lambda}\right)$$

$$= \frac{\left(1.30\times10^{18}\text{ photons/s}\right)\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)}{\pi\left(1.00\times10^{-3}\text{ m}\right)^2}\left(\frac{3.00\times10^{8}\text{ m/s}}{514.5\times10^{-9}\text{ m}}\right)$$

$$= 1.60\times10^{5}\text{ W/m}^2$$

where N is the number of photons per second emitted. Then,

$$E = \sqrt{S/(\varepsilon_0 c)} = \boxed{7760\text{ N/C}}$$

$$\theta = \tan^{-1}\left(\frac{\Delta p_y}{p_x}\right) = \tan^{-1}\left(\frac{\frac{\cancel{h}}{2\pi W}}{\frac{\cancel{h}}{\lambda}}\right) = \tan^{-1}\left(\frac{\lambda}{2\pi W}\right) = \tan^{-1}\left[\frac{633\times10^{-9}\ \text{m}}{2\pi\left(0.200\times10^{-3}\ \text{m}\right)}\right]$$

$$= 0.0289^\circ$$

Therefore, the particles spread out over the range $\boxed{-0.0289^\circ \text{ to } +0.0289^\circ}$.

---

39. SSM WWW ***REASONING*** The uncertainty in the electron's position is $\Delta y = 3.0 \times 10^{-15}$ m. The minimum uncertainty Δp_y in the y component of the electron's momentum is given by the Heisenberg uncertainty principle as $\Delta p_y = h/(4\pi\Delta y)$ (Equation 29.10).

SOLUTION Setting $\Delta y = 3.0 \times 10^{-15}$ m in the relation $\Delta p_y = h/(4\pi\Delta y)$ gives

$$\Delta p_y = \frac{h}{4\pi\Delta y} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(3.0\times10^{-15}\ \text{m}\right)} = \boxed{1.8\times10^{-20}\ \text{kg}\cdot\text{m/s}}$$

---

40. ***REASONING*** Suppose the object is moving along the $+y$ axis. The uncertainty in the object's position is $\Delta y = 2.5$ m. The minimum uncertainty Δp_y in the object's momentum is specified by the Heisenberg uncertainty principle (Equation 29.10) in the form $(\Delta p_y)(\Delta y) = h/(4\pi)$. Since momentum is mass m times velocity v, the uncertainty in the velocity Δv is related to the uncertainty in the momentum by $\Delta v = (\Delta p_y)/m$.

SOLUTION
a. Using the uncertainty principle, we find the minimum uncertainty in the momentum as follows:

$$\left(\Delta p_y\right)\left(\Delta y\right) = \frac{h}{4\pi}$$

$$\Delta p_y = \frac{h}{4\pi\,\Delta y} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(2.5\ \text{m}\right)} = \boxed{2.1\times10^{-35}\ \text{kg}\cdot\text{m/s}}$$

b. For a golf ball this uncertainty in momentum corresponds to an uncertainty in velocity that is given by

$$\Delta v_y = \frac{\Delta p_y}{m} = \frac{2.1\times10^{-35}\ \text{kg}\cdot\text{m/s}}{0.045\ \text{kg}} = \boxed{4.7\times10^{-34}\ \text{m/s}}$$

c. For an electron this uncertainty in momentum corresponds to an uncertainty in velocity that is given by

$$\Delta v_y = \frac{\Delta p_y}{m} = \frac{2.1\times10^{-35}\ \text{kg}\cdot\text{m/s}}{9.11\times10^{-31}\ \text{kg}} = \boxed{2.3\times10^{-5}\ \text{m/s}}$$

SOLUTION The minimum uncertainty Δy in the position of the particle is related to the minimum uncertainty Δp_y in the momentum via the Heisenberg uncertainty principle. To cast this relationship into a form that gives us the desired percentage for the minimum uncertainty in the speed, we note that the minimum uncertainty in the position is specified as the de Broglie wavelength λ. We can then express the de Broglie wavelength in terms of Planck's constant h and the magnitude p_y of the particle's momentum. The magnitude of the momentum is related to the mass m and the speed v_y of the particle.

SOLUTION The percentage minimum uncertainty in the speed is

$$\text{Percentage} = \frac{\Delta v_y}{v_y}\times100\% \qquad (1)$$

According to the Heisenberg uncertainty principle, the minimum uncertainty Δp_y in the momentum and the minimum uncertainty Δy in the position of the particle are related according to

$$\left(\Delta p_y\right)\left(\Delta y\right) = \frac{h}{4\pi} \qquad (29.10)$$

We know that Δy is equal to the de Broglie wavelength $\lambda = h/p_y$ (Equation 29.8), where the magnitude of the momentum is $p_y = mv_y$ (Equation 7.2). Thus, we have

$$\Delta y = \lambda = \frac{h}{p_y} = \frac{h}{mv_y}$$

Substituting this result for Δy into Equation 29.10, we obtain

$$\left(\Delta p_y\right)\left(\Delta y\right) = \left(\Delta p_y\right)\left(\frac{h}{mv_y}\right) = \frac{h}{4\pi} \qquad (2)$$

The last step in our transformation of the uncertainty principle is to realize that $\Delta p_y = \Delta\left(mv_y\right) = m\Delta v_y$, since the mass is constant. Substituting this expression for Δp_y into Equation (2) shows that

Substituting this expression for f into Equation 29.2 gives

$$E = hf = h\frac{c}{\lambda}$$

Applying this result to both sources, we have

$$E_B = h\frac{c}{\lambda_B} \quad \text{and} \quad E_A = h\frac{c}{\lambda_A}$$

Dividing the two expressions gives

$$\frac{E_B}{E_A} = \frac{h\dfrac{c}{\lambda_B}}{h\dfrac{c}{\lambda_A}} = \frac{\lambda_A}{\lambda_B}$$

Using the given value for E_A and the fact that $\lambda_B = 3\lambda_A$ in this result shows that

$$E_B = E_A\frac{\lambda_A}{\lambda_B} = \left(2.1\times10^{-18}\ \text{J}\right)\left(\frac{\lambda_A}{3\lambda_A}\right) = \boxed{7.0\times10^{-19}\ \text{J}}$$

45. ***REASONING*** The de Broglie wavelength λ is related to Planck's constant h and the magnitude p of the particle's momentum. The magnitude of the momentum is related to the mass m and the speed v at which the bacterium is moving. Since the mass and the speed are given, we can calculate the wavelength directly.

SOLUTION The de Broglie wavelength is

$$\lambda = \frac{h}{p} \tag{29.8}$$

The magnitude of the momentum is $p = mv$ (Equation 7.2), which we can substitute into Equation 29.8 to show that the de Broglie wavelength of the bacterium is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(2\times10^{-15}\ \text{kg}\right)\left(0.33\ \text{m/s}\right)} = \boxed{1\times10^{-18}\ \text{m}}$$

46. ***REASONING AND SOLUTION***

a. We know $E = hc/\lambda$ for a photon. The energy of the photon is

$$E = 5.0\ \text{eV}\left(\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}\right) = 8.0\times10^{-19}\ \text{J}$$

The wavelength is

$$\lambda = \frac{hc}{E} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{8.0\times10^{-19}\ \text{J}} = \boxed{2.5\times10^{-7}\ \text{m}}$$

b. The speed of the 5.0-eV electron is

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\left(8.0\times10^{-19}\ \text{J}\right)}{9.11\times10^{-31}\ \text{kg}}} = 1.3\times10^{6}\ \text{m/s}$$

The de Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.3\times10^{6}\ \text{m/s}\right)} = \boxed{5.6\times10^{-10}\ \text{m}}$$

47. ***REASONING AND SOLUTION*** In the first case, the energy of the incident photon is given by Equation 29.3 as

$$hf = \text{KE}_{\text{max}} + W_0 = 0.68\ \text{eV} + 2.75\ \text{eV} = 3.43\ \text{eV}$$

In the second case, a rearrangement of Equation 29.3 yields

$$\text{KE}_{\text{max}} = hf - W_0 = 3.43\ \text{eV} - 2.17\ \text{eV} = \boxed{1.26\ \text{eV}}$$

48. ***REASONING*** The speed v of a particle is related to the magnitude p of its momentum by $v = p/m$ (Equation 7.2). The magnitude of the momentum is related to the particle's de Broglie wavelength λ by $p = h/\lambda$ (Equation 29.8), where h is Planck's constant. Thus, the speed of a particle can be expressed as $v = h/(m\lambda)$. We will use this relation to find the speed of the proton.

SOLUTION The speeds of the proton and electron are

$$v_{\text{proton}} = \frac{h}{m_{\text{proton}}\lambda_{\text{proton}}} \quad \text{and} \quad v_{\text{electron}} = \frac{h}{m_{\text{electron}}\lambda_{\text{electron}}}$$

Dividing the first equation by the second equation, and noting that $\lambda_{\text{electron}} = \lambda_{\text{proton}}$, we obtain

$$\frac{v_{\text{proton}}}{v_{\text{electron}}} = \frac{m_{\text{electron}}\lambda_{\text{electron}}}{m_{\text{proton}}\lambda_{\text{proton}}} = \frac{m_{\text{electron}}}{m_{\text{proton}}}$$

Using values for m_{electron} and m_{proton} taken from the inside of the front cover, we find that the speed of the proton is

$$v_{\text{proton}} = v_{\text{electron}}\left(\frac{m_{\text{electron}}}{m_{\text{proton}}}\right) = \left(4.50\times10^{6}\text{ m/s}\right)\left(\frac{9.11\times10^{-31}\text{ kg}}{1.67\times10^{-27}\text{ kg}}\right) = \boxed{2.45\times10^{3}\text{ m/s}}$$

---

49. ***REASONING*** The width of the central bright fringe in the diffraction patterns will be identical when the electrons have the same de Broglie wavelength as the wavelength of the photons in the red light. The de Broglie wavelength of one electron in the beam is given by Equation 29.8, $\lambda_{\text{electron}} = h/p$, where $p = mv$.

SOLUTION Following the reasoning described above, we find

$$\lambda_{\text{red light}} = \lambda_{\text{electron}}$$

$$\lambda_{\text{red light}} = \frac{h}{m_{\text{electron}} v_{\text{electron}}}$$

Solving for the speed of the electron, we have

$$v_{\text{electron}} = \frac{h}{m_{\text{electron}}\lambda_{\text{red light}}} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{(9.11\times10^{-31}\text{ kg})(661\times10^{-9}\text{ m})} = \boxed{1.10\times10^{3}\text{ m/s}}$$

---

50. ***REASONING*** We will first calculate the potential energy of the system at each of the two separations, and then find the energy difference for the two configurations. Since the electric potential energy lost by the system is carried off by a photon that is emitted during the process, the energy difference must be equal to the energy of the photon. The wavelength of the photon can then by found using Equation 29.2 with Equation 16.1: $E = hc/\lambda$.

SOLUTION The initial potential energy of the system is (see Equations 19.3 and 19.6)

$$\text{EPE}_1 = eV_1 = e\left(\frac{kq}{r_1}\right)$$

$$= (1.6\times10^{-19}\text{ C})\left[\frac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(8.30\times10^{-6}\text{ C})}{0.420\text{ m}}\right] = 2.84\times10^{-14}\text{ J}$$

The final potential energy is

$$\text{EPE}_2 = eV_2 = (1.6\times10^{-19}\text{ C})\left[\frac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(8.30\times10^{-6}\text{ C})}{1.58\text{ m}}\right] = 7.56\times10^{-15}\text{ J}$$

The energy difference, and therefore the energy of the emitted photon, is

$$\Delta E = \text{EPE}_1 - \text{EPE}_2 = 2.84\times10^{-14}\text{ J} - 7.56\times10^{-15}\text{ J} = 2.08\times10^{-14}\text{ J}$$

The wavelength of this photon is

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{2.08\times10^{-14}\text{ J}} = \boxed{9.56\times10^{-12}\text{ m}}$$

---

51. ***REASONING*** Since the net external force acting on the system (the photon and the electron) is zero, the conservation of linear momentum applies. In addition, there are no nonconservative forces, so the conservation of total energy applies as well. Since the photon scatters at an angle of $\theta = 180.0°$ in Figure 29.10, the collision is "head-on." Thus, the motion takes place entirely along the horizontal direction, which we will take as the x axis, with the right as being the positive direction.

The conservation of linear momentum gives rise to Equation 29.7, which relates the difference $\lambda' - \lambda$ between the scattered and incident X-ray photon wavelengths to the scattering angle θ of the electron as

$$\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta) = \frac{h}{mc}(1-\cos 180.0°) = \frac{2h}{mc} \tag{1}$$

The conservation of total energy is written as

$$\underbrace{\frac{hc}{\lambda}}_{\substack{\text{Energy}\\\text{of incident}\\\text{photon}}} + \underbrace{0}_{\substack{\text{Initial}\\\text{kinetic}\\\text{energy}\\\text{of electron}}} = \underbrace{\frac{hc}{\lambda'}}_{\substack{\text{Energy}\\\text{of scattered}\\\text{photon}}} + \underbrace{\tfrac{1}{2}mv^2}_{\substack{\text{Final}\\\text{kinetic}\\\text{energy}\\\text{of electron}}} \tag{2}$$