SOLUTION Substituting $V_A = 0$ V and $q_0 = -e$ into $W_{AB} = -q_0(V_B - V_A)$ (Equation 19.4) yields

$$W_{AB} = -q_0(V_B - V_A) = -e(V_B - 0\text{ V}) = -eV_B \tag{2}$$

Substituting Equation (1) into Equation (2), we obtain

$$W_{AB} = -eV_B = -\frac{kZe^2}{r_B} = -\frac{29(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\text{ C})^2}{4.8\times10^{-15}\text{ m}}$$

Converting this result to electron volts with the equivalence 1 eV = 1.6×10^{-19} J, we find that

$$W_{AB} = \left[-\frac{29(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\text{ C})^2}{4.8\times10^{-15}\text{ m}}\right]\left(\frac{1\text{ eV}}{1.6\times10^{-19}\text{ J}}\right) = \boxed{-8.7\times10^6\text{ eV}}$$

---

7. SSM WWW ***REASONING AND SOLUTION***

a. The longest wavelength in the Pfund series occurs for the transition $n = 6$ to $n = 5$, so that according to Equation 30.14 with $Z = 1$, we have

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right) = (1.097\times10^7\text{ m}^{-1})\left(\frac{1}{5^2} - \frac{1}{6^2}\right) \quad \text{or} \quad \boxed{\lambda = 7458\text{ nm}}$$

b. The shortest wavelength occurs when $1/n^2 = 0$, so that

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right) = (1.097\times10^7\text{ m}^{-1})\left(\frac{1}{5^2}\right) \quad \text{or} \quad \boxed{\lambda = 2279\text{ nm}}$$

c. The lines in the Pfund series occur in the | infrared region |.

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8. ***REASONING*** The atomic number for helium is $Z = 2$. The ground state is the $n = 1$ state, the first excited state is the $n = 2$ state, and the second excited state is the $n = 3$ state. With $Z = 2$ and $n = 3$, we can use Equation 30.10 to find the radius of the ion.

SOLUTION The radius of the second excited state is

$$r_3 = (5.29\times10^{-11}\text{ m})\frac{n^2}{Z} = (5.29\times10^{-11}\text{ m})\frac{3^2}{2} = \boxed{2.38\times10^{-10}\text{ m}} \tag{30.10}$$

---

9. ***REASONING*** According to the Bohr model, the energy (in joules) of the nth orbit of an atom containing a single electron is

$$E_n = -(2.18\times10^{-18}\text{ J})\frac{Z^2}{n^2} \tag{30.12}$$

where Z is the atomic number of the atom. The ratio of the energies of the two atoms can be obtained directly by using this relation.

SOLUTION Taking the ratio of the energy $E_{n,\,\text{Be}^{3+}}$ of the nth orbit of a beryllium atom $(Z_{\text{Be}^{3+}} = 4)$ to the energy $E_{n,\,\text{H}}$ of the nth orbit of a hydrogen ($Z_{\text{H}} = 1$) atom gives

$$\frac{E_{n,\,\text{Be}^{3+}}}{E_{n,\,\text{H}}} = \frac{-(2.18\times10^{-18}\text{ J})\dfrac{Z_{\text{Be}^{3+}}^2}{n^2}}{-(2.18\times10^{-18}\text{ J})\dfrac{Z_{\text{H}}^2}{n^2}} = \frac{Z_{\text{Be}^{3+}}^2}{Z_{\text{H}}^2} = \frac{(4)^2}{(1)^2} = \boxed{16}$$

---

10. ***REASONING***

a. The total energy E_n for a single electron in the n^{th} state is given by

$$E_n = -(13.6\text{ eV})\frac{Z^2}{n^2} \tag{30.13}$$

where $Z = 2$ for helium. The minimum amount of energy required to remove the electron from the ground state ($n = 1$) is that needed to move the electron into the state for which $n = 2$. This amount equals the difference between the two energy levels.

b. The ionization energy defined as the minimum amount of energy required to remove the electron from the $n = 1$ orbit to the highest possible excited state $(n = \infty)$.

SOLUTION

a. The minimum amount of energy required to remove the electron from the ground state ($n = 1$) and move it into the state for which $n = 2$ is

$$\text{Minimum energy} = E_2 - E_1 = \frac{-(13.6\text{ eV})(2)^2}{2^2} - \left[\frac{-(13.6\text{ eV})(2)^2}{1^2}\right] = \boxed{40.8\text{ eV}}$$

b. The ionization energy is the difference between the ground-state energy ($n = 1$) and the energy in the highest possible excited state $(n = \infty)$. Thus,

$$\text{Ionization energy} = E_\infty - E_1 = \frac{-(13.6\text{ eV})(2)^2}{(\infty)^2} - \left[\frac{-(13.6\text{ eV})(2)^2}{1^2}\right] = \boxed{54.4\text{ eV}}$$

---

11. SSM ***REASONING*** According to Equation 30.14, the wavelength λ emitted by the hydrogen atom when it makes a transition from the level with n_i to the level with n_f is given by

$$\frac{1}{\lambda} = \frac{2\pi^2 mk^2 e^4}{h^3 c}(Z^2)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \quad \text{with} \quad n_i, n_f = 1, 2, 3, \ldots \text{ and } n_i > n_f$$

where $2\pi^2 mk^2 e^4/(h^3 c) = 1.097\times10^7\ \text{m}^{-1}$ and $Z = 1$ for hydrogen. Once the wavelength for the particular transition in question is determined, Equation 29.2 ($E = hf = hc/\lambda$) can be used to find the energy of the emitted photon.

SOLUTION In the Paschen series, $n_f = 3$. Using the above expression with $Z = 1$, $n_i = 7$ and $n_f = 3$, we find that

$$\frac{1}{\lambda} = \left(1.097\times10^7\ \text{m}^{-1}\right)\left(1^2\right)\left(\frac{1}{3^2} - \frac{1}{7^2}\right) \qquad \text{or} \qquad \lambda = 1.005\times10^{-6}\ \text{m}$$

The photon energy is

$$E = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{1.005\times10^{-6}\ \text{m}} = \boxed{1.98\times10^{-19}\ \text{J}}$$

---

12. ***REASONING*** The ionization energy for a given state is the energy needed to remove the electron completely from the atom. The removed electron has no kinetic energy and no electric potential energy, so its total energy is zero.

The ionization energy for a given excited state is less than the ionization energy for the ground state. In the excited state the electron already has part of the energy necessary to achieve ionization, so less energy is required to ionize the atom from the excited state than from the ground state.

SOLUTION

a. The energy of the n^{th} state in the hydrogen atom is given by Equation 30.13 as $E_n = -(13.6\ \text{eV})Z^2/n^2$. When $n = \infty$, $E_\infty = 0\ \text{J}$, and when $n = 4$, $E_4 = -(13.6\ \text{eV})(1)^2/4^2 = -0.850\ \text{eV}$. The difference in energies between these two states is the ionization energy:

$$\text{Ionization energy} = E_\infty - E_4 = \boxed{0.850\ \text{eV}}$$

b. In the same manner, it can be shown that the ionization energy for the $n = 1$ state is 13.6 eV. The ratio of the ionization energies is

$$\frac{0.850\ \text{eV}}{13.6\ \text{eV}} = \boxed{0.0625}$$

---

13. ***REASONING*** Since the atom emits two photons as it returns to the ground state, one is emitted when the electron falls from $n = 3$ to $n = 2$, and the other is emitted when it subsequently drops from $n = 2$ to $n = 1$. The wavelengths of the photons emitted during these transitions are given by Equation 30.14 with the appropriate values for the initial and final numbers, n_i and n_f.

SOLUTION The wavelengths of the photons are

n = 3 to n = 2
$$\frac{1}{\lambda} = \left(1.097\times10^7\ \text{m}^{-1}\right)(1)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.524\times10^6\ \text{m}^{-1} \qquad (30.14)$$

$$\lambda = \boxed{6.56\times10^{-7}\ \text{m}}$$

n = 2 to n = 1
$$\frac{1}{\lambda} = \left(1.097\times10^7\ \text{m}^{-1}\right)(1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 8.228\times10^6\ \text{m}^{-1} \qquad (30.14)$$

$$\lambda = \boxed{1.22\times10^{-7}\ \text{m}}$$

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14. ***REASONING*** The Bohr expression as it applies to any one-electron species of atomic number Z, is given by Equation 30.13: $E_n = -(13.6\ \text{eV})(Z^2/n^2)$. For certain values of the quantum number n, this expression predicts equal electron energies for singly ionized helium He^+ ($Z = 2$) and doubly ionized lithium Li^+ ($Z = 3$). As stated in the problem, the quantum number n is different for the equal energy states for each species.

SOLUTION For equal energies, we can write

$$(E_n)_{\text{He}} = (E_n)_{\text{Li}} \qquad \text{or} \qquad -(13.6\ \text{eV})\frac{Z_{\text{He}}^2}{n_{\text{He}}^2} = -(13.6\ \text{eV})\frac{Z_{\text{Li}}^2}{n_{\text{Li}}^2}$$

Simplifying, this becomes

$$\frac{Z_{\text{He}}^2}{n_{\text{He}}^2} = \frac{Z_{\text{Li}}^2}{n_{\text{Li}}^2} \qquad \text{or} \qquad \frac{4}{n_{\text{He}}^2} = \frac{9}{n_{\text{Li}}^2}$$

Thus,

$$n_{\text{He}} = n_{\text{Li}}\sqrt{\frac{4}{9}} = n_{\text{Li}}\left(\frac{2}{3}\right)$$

Therefore, the value of the helium energy level is equal to the lithium energy level for any value of n_{He} that is two-thirds of n_{Li}. For quantum numbers less than or equal to 9, an equality in energy levels will occur for $n_{\text{He}} = 2, 4, 6$ corresponding to $n_{\text{Li}} = 3, 6, 9$. The results are summarized in the following table.

n_{He}	n_{Li}	Energy
2	3	–13.6 eV
4	6	–3.40 eV
6	9	–1.51 eV

15. ***REASONING*** In the Bohr model of the hydrogen atom the total energy E_n of the electron is given in electron volts by Equation 30.13 and the orbital radius r_n is given in meters by Equation 30.10:

$$E_n = \frac{-13.6}{n^2} \quad \text{and} \quad r_n = \left(5.29\times10^{-11}\right)n^2$$

Solving the radius equation for n^2 and substituting the result into the energy equation gives

$$E_n = \frac{-13.6}{r_n/\left(5.29\times10^{-11}\right)} = \frac{(-13.6)\left(5.29\times10^{-11}\right)}{r_n}$$

Thus, the energy is inversely proportional to the radius, and it is on this fact that we base our solution.

SOLUTION We know that the radius of orbit B is sixteen times greater than the radius of orbit A. Since the total energy is inversely proportional to the radius, it follows that the total energy of the electron in orbit B is one-sixteenth of the total energy in orbit A:

$$E_{\text{B}} = \frac{E_{\text{A}}}{16.0} = \frac{-3.40\text{ eV}}{16.0} = \boxed{-0.213\text{ eV}}$$

16. ***REASONING*** We expect $Z_{\text{effective}}$ to be greater than one. Since the orbiting "inner" electrons spend part of the time on the side of the nucleus opposite to the outermost electron, they are not always between the nucleus and the outermost electron. Therefore, they are sometimes farther away from the outermost electron than the nuclear protons are. As a result, the repulsive force that they exert on the outermost electron sometimes has a magnitude less than the attractive force exerted by the nuclear protons. The net effect of this is that the 10 "inner" electrons only partially shield the outermost electron from the attractive force of 10 nuclear protons, and it follows that $Z_{\text{effective}}$ is greater than one.

In the Bohr model the orbital radius r_n is

$$r_n = \left(5.29\times10^{-11}\text{ m}\right)\frac{n^2}{Z} \tag{30.10}$$

where n is the principal quantum number. The radius is inversely proportional to Z. Therefore, we expect that the radius calculated with $Z = 11$ will be less than the radius calculated with $Z = Z_{\text{effective}}$, where $Z_{\text{effective}} > 1$, but less than 11. This makes sense, because 11 protons would pull the outermost electron in more tightly than 1 or 2 protons would.

SOLUTION

a. According to the Bohr model, the total energy E of the electron is

$$E_n = -(13.6\text{ eV})\frac{Z^2}{n^2} \tag{30.13}$$

To remove an electron with this energy from an atom, it is necessary to give the electron positive energy of a magnitude equal to that in Equation 30.13. This energy is the ionization energy $E_{\text{ionization}}$. Then, the electron will be raised from the nth level to the $n = \infty$ level, where the total energy of the electron is zero and the electron is separated from the atom. Thus, with $Z = Z_{\text{effective}}$ and $n = 3$ for the outermost electron in a sodium atom ($1s^2\ 2s^2\ 2p^6\ 3s^1$) we have

$$E_{\text{ionization}} = (13.6\text{ eV})\frac{Z_{\text{effective}}^2}{3^2} \quad \text{or} \quad Z_{\text{effective}} = \sqrt{\frac{3^2 E_{\text{ionization}}}{13.6\text{ eV}}} = \sqrt{\frac{3^2(5.1\text{ eV})}{13.6\text{ eV}}} = \boxed{1.8}$$

As expected, $Z_{\text{effective}}$ is less than 11 and greater than 1.

b. Using Equation 30.10 for the radius with $Z = 11$ and $Z = Z_{\text{effective}} = 1.8$, we obtain

[$Z = 11$]

$$r_3 = \left(5.29\times10^{-11}\text{ m}\right)\frac{3^2}{11} = \boxed{4.3\times10^{-11}\text{ m}}$$

[$Z = 1.8$] $r_3 = \left(5.29\times10^{-11}\ \text{m}\right)\dfrac{3^2}{1.8} = \boxed{2.6\times10^{-10}\ \text{m}}$

As expected, the radius is smaller when $Z = 11$.

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17. SSM ***REASONING*** A wavelength of 410.2 nm is emitted by the hydrogen atoms in a high-voltage discharge tube. This transition lies in the visible region (380–750 nm) of the hydrogen spectrum. Thus, we can conclude that the transition is in the Balmer series and, therefore, that $n_f = 2$. The value of n_i can be found using Equation 30.14, according to which the Balmer series transitions are given by

$$\frac{1}{\lambda} = R\left(1^2\right)\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right) \qquad n = 3, 4, 5, \ldots$$

This expression may be solved for n_i for the energy transition that produces the given wavelength.

SOLUTION Solving for n_i, we find that

$$n_i = \frac{1}{\sqrt{\dfrac{1}{2^2} - \dfrac{1}{R\lambda}}} = \frac{1}{\sqrt{\dfrac{1}{2^2} - \dfrac{1}{(1.097\times10^7\ \text{m}^{-1})(410.2\times10^{-9}\ \text{m})}}} = 6$$

Therefore, the initial and final states are identified by $\boxed{n_i = 6 \text{ and } n_f = 2}$.

---

18. ***REASONING*** The energy levels and radii of a hydrogenic species of atomic number Z are given by Equations 30.13 and 30.10, respectively: $E_n = -(13.6\ \text{eV})(Z^2/n^2)$ and $r_n = (5.29\times10^{-11}\text{m})(n^2/Z)$. We can use Equation 30.13 to find the value of Z for the unidentified ionized atom and then calculate the radius of the $n = 5$ orbit using Equation 30.10.

SOLUTION Solving Equation 30.13 for atomic number Z of the unknown species, we have

$$Z = \sqrt{\frac{E_n n^2}{-13.6\ \text{eV}}} = \sqrt{\frac{(-30.6\ \text{eV})(2)^2}{-13.6\ \text{eV}}} = 3$$

Therefore, the radius of the $n = 5$ orbit is

$$r_5 = (5.29\times10^{-11}\ \text{m})\left(\frac{5^2}{3}\right) = \boxed{4.41\times10^{-10}\ \text{m}}$$

---

19. SSM ***REASONING*** Singly ionized helium, He^+, is a hydrogen-like species with $Z = 2$. The wavelengths of the series of lines produced when the electron makes a transition from higher energy levels into the $n_f = 4$ level are given by Equation 30.14 with $Z = 2$ and $n_f = 4$:

$$\frac{1}{\lambda} = \left(1.097\times10^7\ \text{m}^{-1}\right)(2^2)\left(\frac{1}{4^2} - \frac{1}{n_i^2}\right)$$

SOLUTION Solving this expression for n_i gives

$$n_i = \left[\frac{1}{4^2} - \frac{1}{4\lambda(1.097\times10^7\ \text{m}^{-1})}\right]^{-1/2}$$

Evaluating this expression at the limits of the range for λ, we find that $n_i = 19.88$ for $\lambda = 380$ nm, and $n_i = 5.58$ for $\lambda = 750$ nm. Therefore, the values of n_i for energy levels from which the electron makes the transitions that yield wavelengths in the range between 380 nm and 750 nm are $\boxed{6 \le n_i \le 19}$.

---

20. ***REASONING*** For each species, the wavelengths appearing in the line spectra can be calculated using Equation 30.14:

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The shortest wavelength for a given series of lines occurs when an electron is in an initial energy level with a principal quantum number of $n_i = \infty$. Therefore, $1/n_i^2 = 1/\infty = 0$, and the shortest wavelength is given by

$$\frac{1}{\lambda} = \frac{RZ^2}{n_f^2}$$

In this expression, the value of n_f is the same for the series for Li^{2+} and for Be^{3+}. The term R is the Rydberg constant and is also the same for both ionic species. Thus, rearranging this equation gives

$$\omega_n = \frac{nh}{2\pi m r_n^2} = \frac{nh}{2\pi m\left(5.29\times10^{-11}\text{ m}\right)^2\left(n^2\right)^2} = \frac{h}{2\pi m\left(5.29\times10^{-11}\text{ m}\right)^2 n^3} \quad (4)$$

To find the ground-state angular speed of the electron in revolutions per second, we substitute $n = 1$ into Equation (4) and multiply by the appropriate conversion factor:

$$\omega_1 = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)}{2\pi\left(9.11\times10^{-31}\text{ kg}\right)\left(5.29\times10^{-11}\text{ m}\right)^2(1)^3}\times\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{6.59\times10^{15}\text{ rev/s}}$$

b. When the electron is in the first excited state, the value of the principal quantum number is $n = 2$, so by Equation (4) we have that

$$\omega_2 = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)}{2\pi\left(9.11\times10^{-31}\text{ kg}\right)\left(5.29\times10^{-11}\text{ m}\right)^2(2)^3}\times\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{8.23\times10^{14}\text{ rev/s}}$$

---

23. ***REASONING*** The orbital quantum number ℓ has values of 0, 1, 2, ..., $(n-1)$, according to the discussion in Section 30.5. Since $\ell = 5$, we can conclude, therefore, that $n \geq 6$. This knowledge about the principal quantum number n can be used with Equation 30.13, $E_n = -(13.6\text{ eV})Z^2/n^2$, to determine the smallest value for the total energy E_n.

SOLUTION The smallest value of E_n (i.e., the most negative) occurs when $n = 6$. Thus, using $Z = 1$ for hydrogen, we find

$$E_n = -(13.6\text{ eV})\frac{Z^2}{n^2} = -(13.6\text{ eV})\frac{1^2}{6^2} = \boxed{-0.378\text{ eV}}$$

---

24. ***REASONING*** The orbital quantum number ℓ can have any integer value from 0 up to $n-1$, so that it must be less than the principle quantum number n and it cannot be a negative number. If for example, $n = 4$, ℓ can have only the values 0, 1, 2, and 3.

The magnetic quantum number m_ℓ can only have integer values, including 0, from $-\ell$ to $+\ell$. Thus, it can have a negative value. For instance, if $\ell = 2$, m_ℓ can have the values –2, –1, 0 ,+1, and +2.

SOLUTION Of the five states listed in the table, three are not possible. The ones that are not possible, and the reasons they are not possible, are:

State	Reason
(a)	The quantum number ℓ must be less than n
(c)	The quantum number m_ℓ must be less than or equal to ℓ.
(d)	The quantum number ℓ cannot be negative.

---

25. ***REASONING***

a. The ground state is the $n = 1$ state, the first excited state is the $n = 2$ state, and the second excited state is the $n = 3$ state. The total energy (in eV) of a hydrogen atom in the $n = 3$ state is given by Equation 30.13.

b. According to quantum mechanics, the magnitude L of the angular momentum is given by Equation 30.15 as $L = \sqrt{\ell(\ell+1)}\left(h/2\pi\right)$, where ℓ is the orbital quantum number. The discussion in Section 30.5 indicates that the maximum value that ℓ can have is one less than the principal quantum number, so that $\ell_{\text{max}} = n - 1$.

c. Equation 30.16 gives the z-component L_z of the angular momentum as $L_z = m_\ell\left(h/2\pi\right)$, where m_ℓ is the magnetic quantum number. According to the discussion in Section 30.5, the maximum value that m_ℓ can attain is when it is equal to the orbital quantum number, which is ℓ_{max}.

SOLUTION

a. The total energy of the hydrogen atom is given by Equation 30.13. Using $n = 3$, we have

$$E_3 = -\frac{(13.6\text{ eV})(1)^2}{3^2} = \boxed{-1.51\text{ eV}}$$

b. The maximum orbital quantum number is $\ell_{\text{max}} = n - 1 = 3 - 1 = 2$. The maximum angular momentum L_{max} has a magnitude given by Equation 30.15:

$$L_{\text{max}} = \sqrt{\ell_{\text{max}}\left(\ell_{\text{max}}+1\right)}\frac{h}{2\pi} = \sqrt{2(2+1)}\frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{2\pi} = \boxed{2.58\times10^{-34}\text{ J}\cdot\text{s}}$$

c. The maximum value for the z-component L_z of the angular momentum is $(\text{with } m_\ell = \ell_{\max} = 2)$

$$L_z = m_\ell \frac{h}{2\pi} = (2)\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi} = \boxed{2.11\times10^{-34}\ \text{J}\cdot\text{s}}$$

---

26. ***REASONING*** The maximum value for the magnetic quantum number is $m_\ell = \ell$; thus, in state A, $\ell = 2$, while in state B, $\ell = 1$. According to the quantum mechanical theory of angular momentum, the magnitude of the orbital angular momentum for a state of given ℓ is $L = \sqrt{\ell(\ell+1)}\,(h/2\pi)$ (Equation 30.15). This expression can be used to form the ratio L_A / L_B of the magnitudes of the orbital angular momenta for the two states.

SOLUTION Using Equation 30.15, we find that

$$\frac{L_A}{L_B} = \frac{\sqrt{2(2+1)}\,\dfrac{h}{2\pi}}{\sqrt{1(1+1)}\,\dfrac{h}{2\pi}} = \sqrt{\frac{6}{2}} = \sqrt{3} = \boxed{1.732}$$

---

27. **SSM** **WWW** ***REASONING*** The values that ℓ can have depend on the value of n, and only the following integers are allowed: $\ell = 0, 1, 2, \ldots (n-1)$. The values that m_ℓ can have depend on the value of ℓ, with only the following positive and negative integers being permitted: $m_\ell = -\ell, \ldots -2, -1, 0, +1, +2, \ldots +\ell$.

SOLUTION Thus, when $n = 6$, the possible values of ℓ are 0, 1, 2, 3, 4, 5. Now when $m_\ell = 2$, the possible values of ℓ are 2, 3, 4, 5, ... These two series of integers overlap for the integers 2, 3, 4, and 5. Therefore, the possible values for the orbital quantum number ℓ that this electron could have are $\boxed{\ell = 2, 3, 4, 5}$.

---

28. ***REASONING*** The greater the magnitude of the magnetic quantum number m_ℓ, the greater the magnitude of the z component L_z of the electron's angular momentum, as we see from $L_z = m_\ell \dfrac{h}{2\pi}$ (Equation 30.16), where $h = 6.626\times10^{-34}$ J·s is Planck's constant. The maximum value of the magnetic quantum number m_ℓ can have is the value of the orbital quantum number ℓ, so we know that $m_\ell = \ell$. The angular momentum L of the electron can be found from the orbital quantum number ℓ via $L = \sqrt{\ell(\ell+1)}\,\dfrac{h}{2\pi}$ (Equation 30.15).

SOLUTION Squaring both sides of Equation 30.15 and solving for the quantity $\ell(\ell+1)$, we obtain

$$L^2 = \ell(\ell+1)\left(\frac{h}{2\pi}\right)^2 \qquad \text{or} \qquad \ell(\ell+1) = \left(\frac{2\pi L}{h}\right)^2 \tag{1}$$

The orbital quantum number ℓ can only take on nonnegative integral values: $\ell = 0, 1, 2, 3, \ldots$. The quantity $\ell(\ell+1)$ in Equation (1), therefore, is a product of two successive integers, and must itself be an integer. Substituting $L = 8.948\times10^{-34}$ J·s into Equation (1) yields

$$\ell(\ell+1) = \left[\frac{2\pi\left(8.948\times10^{-34}\ \text{J}\cdot\text{s}\right)}{6.626\times10^{-34}\ \text{J}\cdot\text{s}}\right]^2 = 72 \tag{2}$$

The only positive integer ℓ that satisfies Equation (2) is 8: $\ell(\ell+1) = 8(8+1) = 8(9) = 72$. Substituting $m_\ell = \ell = 8$ into $L_z = m_\ell \dfrac{h}{2\pi}$ (Equation 30.16), we obtain the maximum possible magnitude for the z component of the electron's angular momentum:

$$L_z = 8\frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)}{2\pi} = \boxed{8.436\times10^{-34}\ \text{J}\cdot\text{s}}$$

---

29. **SSM** **WWW** ***REASONING AND SOLUTION***

a. For the angular momentum, Bohr's value is given by Equation 30.8, with $n = 1$,

$$L_n = \frac{nh}{2\pi} = \boxed{\frac{h}{2\pi}}$$

According to quantum theory, the angular momentum is given by Equation 30.15. For $n = 1$, $\ell = 0$

$$L = \sqrt{\ell(\ell+1)}\left(\frac{h}{2\pi}\right) = \sqrt{0(0+1)}\left(\frac{h}{2\pi}\right) = \boxed{0\ \text{J}\cdot\text{s}}$$

b. For $n = 3$; Bohr theory gives

$$L_n = \frac{nh}{2\pi} = \boxed{\frac{3h}{2\pi}}$$

while quantum mechanics gives

[$n = 3, \ell = 0$] $L = \sqrt{\ell(\ell+1)}\left(\frac{h}{2\pi}\right) = \sqrt{0(0+1)}\left(\frac{h}{2\pi}\right) = \boxed{0\ \text{J}\cdot\text{s}}$

[$n = 3, \ell = 1$] $L = \sqrt{\ell(\ell+1)}\left(\frac{h}{2\pi}\right) = \sqrt{1(1+1)}\left(\frac{h}{2\pi}\right) = \boxed{\frac{\sqrt{2}h}{2\pi}}$

[$n = 3, \ell = 2$] $L = \sqrt{\ell(\ell+1)}\left(\frac{h}{2\pi}\right) = \sqrt{2(2+1)}\left(\frac{h}{2\pi}\right) = \boxed{\frac{\sqrt{6}h}{2\pi}}$

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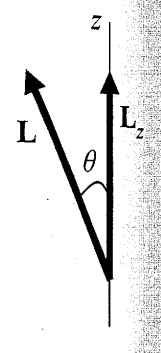
30. ***REASONING*** From the drawing, we see that the angle θ is related to the magnitude L of the orbital angular momentum and the magnitude L_z of the z component of the orbital angular momentum by



$$\theta = \cos^{-1}\left(\frac{L_z}{L}\right) \qquad (1.5)$$

The magnitude L of the orbital angular momentum is given by $L = \sqrt{\ell(\ell+1)}\,\frac{h}{2\pi}$ (Equation 30.15), where ℓ is the orbital quantum number and $h = 6.63\times10^{-34}$ J·s is Planck's constant. The orbital quantum number can have any integer value from $\ell = 0$ to $\ell = n-1$, where $n = 5$ is the principal quantum number. The magnitude L_z of the z component of the orbital angular momentum is given by $L_z = m_\ell \frac{h}{2\pi}$ (Equation 30.16), where m_ℓ is the magnetic quantum number, an integer ranging in value from $-\ell$ to ℓ.

For a given magnitude L of the orbital angular momentum (and, hence, a given value of the orbital quantum number ℓ), the smaller the angle θ becomes, the greater the magnitude L_z of the z component of the orbital angular momentum. Therefore, we will determine the smallest value of θ by choosing the greatest possible value of the magnetic quantum number: $m_\ell = \ell$.

SOLUTION Substituting $m_\ell = \ell$ into $L_z = m_\ell \frac{h}{2\pi}$ (Equation 30.16), we obtain

$$L_z = m_\ell \frac{h}{2\pi} = \ell \frac{h}{2\pi} \qquad (1)$$

Substituting $L = \sqrt{\ell(\ell+1)}\,\frac{h}{2\pi}$ and Equation (1) into Equation (1.5) yields

$$\theta = \cos^{-1}\left(\frac{L_z}{L}\right) = \cos^{-1}\left(\frac{\ell \frac{h}{2\pi}}{\sqrt{\ell(\ell+1)}\,\frac{h}{2\pi}}\right) = \cos^{-1}\left(\frac{\ell}{\sqrt{\ell(\ell+1)}}\right) = \cos^{-1}\left(\sqrt{\frac{\ell}{\ell+1}}\right) \qquad (2)$$

We must choose the value of the orbital quantum number ℓ such that the angle θ in Equation (2) is as small as possible. This means making the quantity inside the square root as *large* as possible. Given that ℓ is an integer, we see that the quantity in the square root of Equation (2) will be of the form $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$. Therefore, to obtain the smallest value for θ, we must choose the greatest possible value for the orbital quantum number ℓ, which is $\ell = n-1 = 5-1 = 4$. Substituting this value into Equation (2), we obtain

$$\theta = \cos^{-1}\left(\sqrt{\frac{\ell}{\ell+1}}\right) = \cos^{-1}\left(\sqrt{\frac{4}{4+1}}\right) = \cos^{-1}\left(\sqrt{\frac{4}{5}}\right) = \boxed{26.6^\circ}$$

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31. ***REASONING***

a. The maximum number of electrons that the $n = 1$, $\ell = 0$ subshell can contain is 2 (see Figure 30.15). If the atom is in its ground state, the third electron must go into the $n = 2$, $\ell = 0$ subshell. For this subshell, the magnetic quantum number can only be zero ($m_\ell = 0$), and the electron can have a spin quantum number of either $m_s = +\frac{1}{2}$ or $-\frac{1}{2}$.

b. If the atom is in its first excited state, the third electron must go into the $n = 2$, $\ell = 1$ subshell. For $\ell = 1$, there are three possibilities for m_ℓ: $m_\ell = +1$, 0, and -1. For each value of m_ℓ, the electron can have a spin quantum number of either $m_s = +\frac{1}{2}$ or $-\frac{1}{2}$.

SOLUTION

a. According to the discussion in the ***REASONING*** section, the third electron can have one of two possibilities for the four quantum numbers: $\boxed{n = 2, \ell = 0, m_\ell = 0, m_s = +\frac{1}{2}}$ and $\boxed{n = 2, \ell = 0, m_\ell = 0, m_s = -\frac{1}{2}}$.