

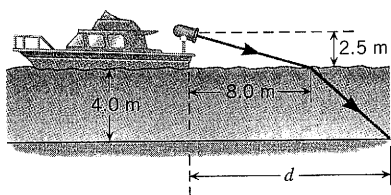
- * **7. Interactive Solution 26.7** at www.wiley.com/college/cutnell offers one model for problems like this one. In a certain time, light travels 6.20 km in a vacuum. During the same time, light travels only 3.40 km in a liquid. What is the refractive index of the liquid?
- * **8.** A flat sheet of ice has a thickness of 2.0 cm. It is on top of a flat sheet of crystalline quartz that has a thickness of 1.1 cm. Light strikes the ice perpendicularly and travels through it and then through the quartz. In the time it takes the light to travel through the two sheets, how far (in centimeters) would it have traveled in a vacuum?

Section 26.2 Snell's Law and the Refraction of Light

9. ssm A light ray in air is incident on a water surface at a 43° angle of incidence. Find (a) the angle of reflection and (b) the angle of refraction.

10. A ray of light impinges from air onto a block of ice ($n = 1.309$) at a 60.0° angle of incidence. Assuming that this angle remains the same, find the difference $\theta_{2, \text{ice}} - \theta_{2, \text{water}}$ in the angles of refraction when the ice turns to water ($n = 1.333$).

11. A spotlight on a boat is 2.5 m above the water, and the light strikes the water at a point that is 8.0 m horizontally displaced from the spotlight (see the drawing). The depth of the water is 4.0 m. Determine the distance d , which locates the point where the light strikes the bottom.



12. Amber ($n = 1.546$) is a transparent brown-yellow fossil resin. An insect, trapped and preserved within the amber, appears to be 2.5 cm beneath the surface when viewed directly from above. How far below the surface is the insect actually located?

13. ssm A beam of light is traveling in air and strikes a material. The angles of incidence and refraction are 63.0° and 47.0° , respectively. Obtain the speed of light in the material.

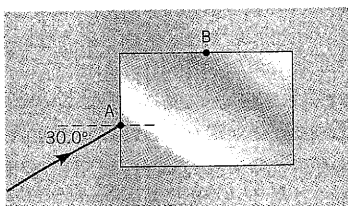
14. A scuba diver, submerged under water, looks up and sees sunlight at an angle of 28.0° from the vertical. At what angle, measured from the vertical, does this sunlight strike the surface of the water?

15. Light in a vacuum is incident on a transparent glass slab. The angle of incidence is 35.0° . The slab is then immersed in a pool of liquid. When the angle of incidence for the light striking the slab is 20.3° , the angle of refraction for the light entering the slab is the same as when the slab was in a vacuum. What is the index of refraction of the liquid?

* **16.** A silver medallion is sealed within a transparent block of plastic. An observer in air, viewing the medallion from directly above, sees the medallion at an apparent depth of 1.6 cm beneath the top surface of the block. How far below the top surface would the medallion appear if the observer (not wearing goggles) and the block were under water?

* **17.** Refer to Figure 26.4b and assume the observer is nearly above the submerged object. For this situation, derive the expression for the apparent depth: $d' = d(n_2/n_1)$, Equation 26.3. (Hint: Use Snell's law of refraction and the fact that the angles of incidence and refraction are small, so $\tan \theta \approx \sin \theta$.)

18. The drawing shows a rectangular block of glass ($n = 1.52$) surrounded by liquid carbon disulfide ($n = 1.63$). A ray of light is incident on the glass at point A with a 30.0° angle of incidence. At what angle of refraction does the ray leave the glass at point B?



* **19. ssm www** In Figure 26.6, suppose that the angle of incidence is $\theta_1 = 30.0^\circ$, the thickness of the glass pane is 6.00 mm, and the refractive index of the glass is $n_2 = 1.52$. Find the amount (in mm) by which the emergent ray is displaced relative to the incident ray.

* **20.** Review Conceptual Example 4 as background for this problem. A man in a boat is looking straight down at a fish in the water directly beneath him. The fish is looking straight up at the man. They are equidistant from the air–water interface. To the man, the fish appears to be 2.0 m beneath his eyes. To the fish, how far above its eyes does the man appear to be?

** **21. ssm** A small logo is embedded in a thick block of crown glass ($n = 1.52$), 3.20 cm beneath the top surface of the glass. The block is put under water, so there is 1.50 cm of water above the top surface of the block. The logo is viewed from directly above by an observer in air. How far beneath the top surface of the water does the logo appear to be?

** **22.** A beaker has a height of 30.0 cm. The lower half of the beaker is filled with water, and the upper half is filled with oil ($n = 1.48$). To a person looking down into the beaker from above, what is the apparent depth of the bottom?

Section 26.3 Total Internal Reflection

23. ssm One method of determining the refractive index of a transparent solid is to measure the critical angle when the solid is in air. If θ_c is found to be 40.5° , what is the index of refraction of the solid?

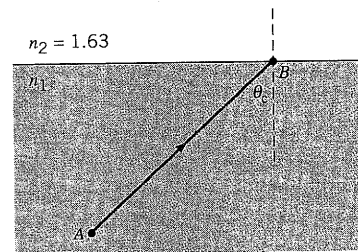
24. A point source of light is submerged 2.2 m below the surface of a lake and emits rays in all directions. On the surface of the lake, directly above the source, the area illuminated is a circle. What is the maximum radius that this circle could have?

25. Interactive Solution 26.25 at www.wiley.com/college/cutnell provides one model for solving problems such as this. A glass block ($n = 1.56$) is immersed in a liquid. A ray of light within the glass hits a glass–liquid surface at a 75.0° angle of incidence. Some of the light enters the liquid. What is the smallest possible refractive index for the liquid?

26. What is the critical angle for light emerging from carbon disulfide into air?

27. Concept Simulation 26.1 at www.wiley.com/college/cutnell illustrates the concepts that are pertinent to this problem. A ray of light is traveling in glass and strikes a glass–liquid interface. The angle of incidence is 58.0° , and the index of refraction of glass is $n = 1.50$. (a) What must be the index of refraction of the liquid such that the direction of the light entering the liquid is not changed? (b) What is the largest index of refraction that the liquid can have, such that none of the light is transmitted into the liquid and all of it is reflected back into the glass?

28. The drawing shows a ray of light traveling from point A to point B, a distance of 4.60 m in a material that has an index of refraction n_1 . At point B, the light encounters a different substance whose index of refraction is $n_2 = 1.63$. The light strikes the interface at the critical angle of $\theta_c = 48.1^\circ$. How much time does it take for the light to travel from A to B?



* **29.** A layer of liquid B floats on liquid A. A ray of light begins in liquid A and undergoes total internal reflection at the interface between the liquids when the angle of incidence exceeds 36.5° . When

Ice $\theta_{2, \text{ice}} = \sin^{-1} \left[\frac{(1.000) \sin 60.0^\circ}{1.309} \right] = 41.4^\circ$

Water $\theta_{2, \text{water}} = \sin^{-1} \left[\frac{(1.000) \sin 60.0^\circ}{1.333} \right] = 40.5^\circ$

The difference in the angles of refraction is $\theta_{2, \text{ice}} - \theta_{2, \text{water}} = 41.4^\circ - 40.5^\circ = \boxed{0.9^\circ}$

11. **REASONING AND SOLUTION** The angle of incidence is found from the drawing to be

$$\theta_1 = \tan^{-1} \left(\frac{8.0 \text{ m}}{2.5 \text{ m}} \right) = 73^\circ$$

Snell's law gives the angle of refraction to be

$$\sin \theta_2 = (n_1/n_2) \sin \theta_1 = (1.000/1.333) \sin 73^\circ = 0.72 \quad \text{or} \quad \theta_2 = 46^\circ$$

The distance d is found from the drawing to be

$$d = 8.0 \text{ m} + (4.0 \text{ m}) \tan \theta_2 = \boxed{12.1 \text{ m}}$$

12. **REASONING AND SOLUTION** Using Equation 26.3, we find

$$d = \left(\frac{n_1}{n_2} \right) d' = \left(\frac{1.546}{1.000} \right) 2.5 \text{ cm} = \boxed{3.9 \text{ cm}}$$

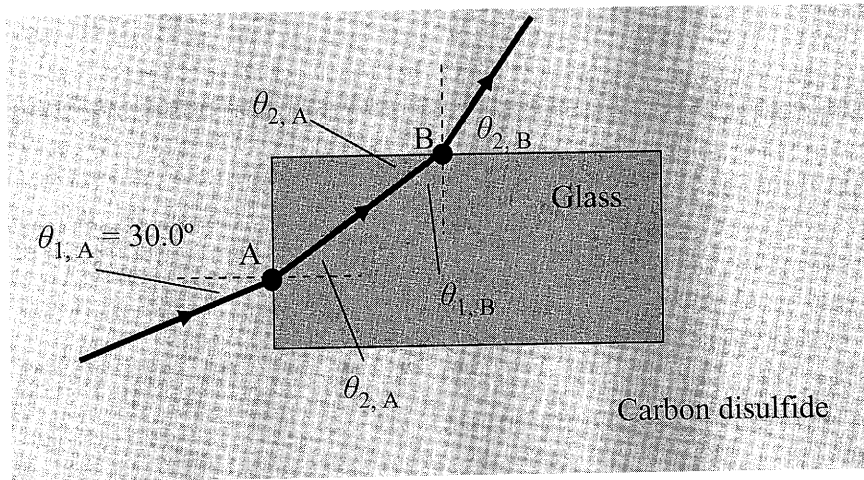
13. **SSM REASONING** We begin by using Snell's law (Equation 26.2: $n_1 \sin \theta_1 = n_2 \sin \theta_2$) to find the index of refraction of the material. Then we will use Equation 26.1, the definition of the index of refraction ($n = c/v$) to find the speed of light in the material.

SOLUTION From Snell's law, the index of refraction of the material is

$$n_2 = \frac{n_1 \sin \theta_1}{\sin \theta_2} = \frac{(1.000) \sin 63.0^\circ}{\sin 47.0^\circ} = 1.22$$

Then, from Equation 26.1, we find that the speed of light v in the material is

18. **REASONING** Snell's law will allow us to calculate the angle of refraction $\theta_{2,B}$ with which the ray leaves the glass at point B, provided that we have a value for the angle of incidence $\theta_{1,B}$ at this point (see the drawing). This angle of incidence is not given, but we can obtain it by considering what happens to the incident ray at point A. This ray is incident at an angle $\theta_{1,A}$ and refracted at an angle $\theta_{2,A}$. Snell's law can be used to obtain $\theta_{2,A}$, the value for which can be combined with the geometry at points A and B to provide the needed value for $\theta_{1,B}$. Since the light ray travels from a material (carbon disulfide) with a higher refractive index toward a material (glass) with a lower refractive index, it is bent away from the normal at point A, as the drawing shows.



SOLUTION Using Snell's law at point B, we have

$$\underbrace{(1.52) \sin \theta_{1,B}}_{\text{Glass}} = \underbrace{(1.63) \sin \theta_{2,B}}_{\text{Carbon disulfide}} \quad \text{or} \quad \sin \theta_{2,B} = \left(\frac{1.52}{1.63} \right) \sin \theta_{1,B} \quad (1)$$

To find $\theta_{1,B}$ we note from the drawing that

$$\theta_{1,B} + \theta_{2,A} = 90.0^\circ \quad \text{or} \quad \theta_{1,B} = 90.0^\circ - \theta_{2,A} \quad (2)$$

We can find $\theta_{2,A}$, which is the angle of refraction at point A, by again using Snell's law:

$$\underbrace{(1.63) \sin \theta_{1,A}}_{\text{Carbon disulfide}} = \underbrace{(1.52) \sin \theta_{2,A}}_{\text{Glass}} \quad \text{or} \quad \sin \theta_{2,A} = \left(\frac{1.63}{1.52} \right) \sin \theta_{1,A}$$

Thus, we have

$$\sin \theta_{2,A} = \left(\frac{1.63}{1.52} \right) \sin \theta_{1,A} = \left(\frac{1.63}{1.52} \right) \sin 30.0^\circ = 0.536 \quad \text{or} \quad \theta_{2,A} = \sin^{-1}(0.536) = 32.4^\circ$$

Using Equation (2), we find that

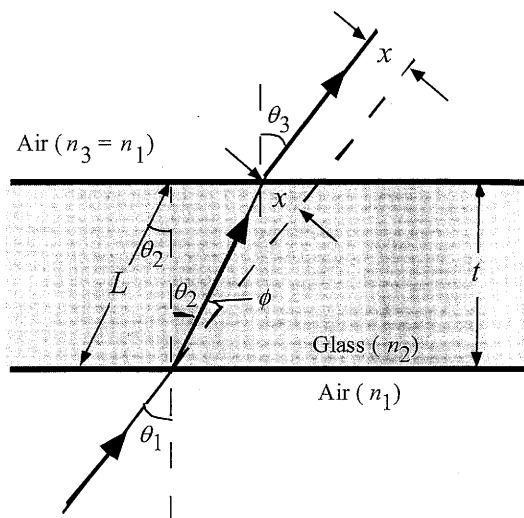
$$\theta_{1,B} = 90.0^\circ - \theta_{2,A} = 90.0^\circ - 32.4^\circ = 57.6^\circ$$

With this value for $\theta_{1,B}$ in Equation (1) we obtain

$$\sin \theta_{2,B} = \left(\frac{1.52}{1.63} \right) \sin \theta_{1,B} = \left(\frac{1.52}{1.63} \right) \sin 57.6^\circ = 0.787 \quad \text{or} \quad \theta_{2,B} = \sin^{-1}(0.787) = \boxed{51.9^\circ}$$

19. **SSM** **WWW** The drawing at the right shows the geometry of the situation using the same notation as that in Figure 26.6. In addition to the text's notation, let t represent the thickness of the pane, let L represent the length of the ray in the pane, let x (shown twice in the figure) equal the displacement of the ray, and let the difference in angles $\theta_1 - \theta_2$ be given by ϕ .

We wish to find the amount x by which the emergent ray is displaced relative to the incident ray. This can be done by applying Snell's law at each interface, and then making use of the geometric and trigonometric relations in the drawing.



SOLUTION If we apply Snell's law (see Equation 26.2) to the bottom interface we obtain $n_1 \sin \theta_1 = n_2 \sin \theta_2$. Similarly, if we apply Snell's law at the top interface where the ray emerges, we have $n_2 \sin \theta_2 = n_3 \sin \theta_3 = n_1 \sin \theta_3$. Comparing this with Snell's law at the bottom face, we see that $n_1 \sin \theta_1 = n_1 \sin \theta_3$, from which we can conclude that $\theta_3 = \theta_1$. Therefore, the emerging ray is parallel to the incident ray.

From the geometry of the ray and thickness of the pane, we see that $L \cos \theta_2 = t$, from which it follows that $L = t / \cos \theta_2$. Furthermore, we see that $x = L \sin \phi = L \sin (\theta_1 - \theta_2)$. Substituting for L , we find

$$x = L \sin(\theta_1 - \theta_2) = \frac{t \sin(\theta_1 - \theta_2)}{\cos \theta_2}$$

Before we can use this expression to determine a numerical value for x , we must find the value of θ_2 . Solving the expression for Snell's law at the bottom interface for θ_2 , we have

As n_{Liquid} decreases, the critical angle decreases. Therefore, n_{Liquid} cannot be less than the value calculated from this equation, in which $\theta_c = 75.0^\circ$ and $n_{\text{Glass}} = 1.56$.

SOLUTION Using Equation 26.4, we find that

$$\sin \theta_c = \frac{n_{\text{Liquid}}}{n_{\text{Glass}}} \quad \text{or} \quad n_{\text{Liquid}} = n_{\text{Glass}} \sin \theta_c = (1.56) \sin 75.0^\circ = \boxed{1.51}$$

26. **REASONING AND SOLUTION** Using Equation 26.4 and taking the refractive index for carbon disulfide from Table 26.1, we obtain

$$\theta_c = \sin^{-1} \left(\frac{1.000}{1.632} \right) = \boxed{37.79^\circ}$$

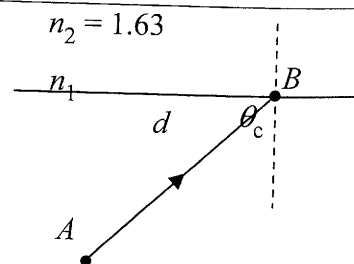
27. **REASONING AND SOLUTION**

- The index of refraction n_2 of the liquid must match that of the glass, or $n_2 = \boxed{1.50}$.
- When none of the light is transmitted into the liquid, the angle of incidence must be equal to or greater than the critical angle. According to Equation 26.4, the critical angle θ_c is given by $\sin \theta_c = n_2/n_1$, where n_2 is the index of refraction of the liquid and n_1 is that of the glass. Therefore,

$$n_2 = n_1 \sin \theta_c = (1.50) \sin 58.0^\circ = \boxed{1.27}$$

If n_2 were larger than 1.27, the critical angle would also be larger, and light would be transmitted from the glass into the liquid. Thus, $n_2 = 1.27$ represents the largest index of refraction of the liquid such that none of the light is transmitted into the liquid.

28. **REASONING** The time it takes for the light to travel from A to B is equal to the distance divided by the speed of light in the substance. The distance is known, and the speed of light v in the substance is equal to the speed of light c in a vacuum divided by the index of refraction n_1 (Equation 26.1). The index of refraction can be obtained by noting that the light is incident at the critical angle θ_c (which is known). According to Equation 26.4, the index of refraction n_1 is related to the critical angle and the index of refraction n_2 by $n_1 = n_2 / \sin \theta_c$.



SOLUTION The time t it takes for the light to travel from A to B is

$$t = \frac{\text{Distance}}{\text{Speed of light in the substance}} = \frac{d}{v} \quad (1)$$

The speed of light v in the substance is related to the speed of light c in a vacuum and the index of refraction n_1 of the substance by $v = c/n_1$ (Equation 26.1). Substituting this expression into Equation (1) gives

$$t = \frac{d}{v} = \frac{d}{\left(\frac{c}{n_1}\right)} = \frac{dn_1}{c} \quad (2)$$

Since the light is incident at the critical angle θ_c , we know that $n_1 \sin \theta_c = n_2$ (Equation 26.4). Solving this expression for n_1 and substituting the result into Equation (2) yields

$$t = \frac{dn_1}{c} = \frac{d \left(\frac{n_2}{\sin \theta_c} \right)}{c} = \frac{(4.60 \text{ m}) \left(\frac{1.63}{\sin 48.1^\circ} \right)}{3.00 \times 10^8 \text{ m/s}} = \boxed{3.36 \times 10^{-8} \text{ s}}$$

29. **REASONING** In the ratio n_B/n_C each refractive index can be related to a critical angle for total internal reflection according to Equation 26.4. By applying this expression to the A-B interface and again to the A-C interface, we will obtain expressions for n_B and n_C in terms of the given critical angles. By substituting these expressions into the ratio, we will be able to obtain a result from which the ratio can be calculated

SOLUTION Applying Equation 26.4 to the A-B interface, we obtain

$$\sin \theta_{c, AB} = \frac{n_B}{n_A} \quad \text{or} \quad n_B = n_A \sin \theta_{c, AB}$$

Applying Equation 26.4 to the A-C interface gives

$$\sin \theta_{c, AC} = \frac{n_C}{n_A} \quad \text{or} \quad n_C = n_A \sin \theta_{c, AC}$$