MULTILAYER NETWORKS

Dangerous liaisons?

Many networks interact with one another by forming multilayer networks, but these structures can lead to large cascading failures. The secret that guarantees the robustness of multilayer networks seems to be in their correlations.

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Natural complex systems evolve according to chance and necessity — trial and error — because they are driven by biological evolution. The expectation is that networks describing natural complex systems, such as the brain and biological networks within the cell, should be robust to random failure. Otherwise, they would have not survived under evolutionary pressure. But many natural networks do not live in isolation; instead they interact with one another to form multilayer networks — and evidence is mounting that random networks of networks are acutely susceptible to failure. Writing in *Nature Physics*, Saulo Reis and colleagues[1] have now identified the key correlations responsible for maintaining robustness within these multilayer networks.

In the past fifteen years, network theory[2,3] has granted solid ground to the expectation that natural networks resist failure. It has also extended the realm of robust systems to man-made self-organized networks that do not obey a centralized design, such as the Internet or the World Wide Web. In fact, it has been shown that many isolated complex biological, technological and social networks are scale free, meaning that their nodes are characterized by a large heterogeneity in terms of the number of connections. Moreover, we now know that the universally observed scale-free property of complex networks is responsible for the robustness of networks in isolation. If we damage the nodes of a large network with probability $1 - p$, where p represents the probability of not damaging the nodes, the critical value ($p = p_c$) required to destroy the connected component of the network comprising an extensive number of nodes is $p_c \approx 0$. In other words, the network always contains a connected component formed by an extensive number of nodes.

The realization that many networks do not live in isolation, but instead interact with one another, is reasonably recent[4,5]. But multilayer networks are everywhere: from infrastructure networks formed by systems such as the power grid, the Internet and financial markets, to multilayer social and communication networks, to the brain and biological networks of the cell. As we cannot understand the living cell if we do not integrate information from all of its biological networks, often the function of one network is interdependent on the function of another network. The same applies in infrastructures and financial markets.

When links are placed between different networks forming multilayer network structures, the robustness of the entire system can be strongly affected[6–8]. In particular, these interconnections can imply interdependencies, meaning that a node in one network is damaged if any of the interdependent nodes in the other layers is damaged. In such cases, the interlinks can trigger cascades of failures propagating back and forth between the two layers and destroying the two networks in an abrupt way characterized by a discontinuous percolation transition at $p = p_c$. In this way, multilayer networks can show a surprising fragility with respect to random damage.

Although this scenario clearly explains the fragility of complex infrastructures such as power grids or financial networks and the interdependent Internet[6], this finding can seem in contradiction with the expected robustness of natural multilayer networks. So what are the properties that keep natural multilayer networks stable? Reis *et al.*[1] solved this puzzle by investigating theoretical models of multilayer networks and studying interconnected brain networks that were found to validate their theory.

Whereas previous results had considered only the effect of random links between the

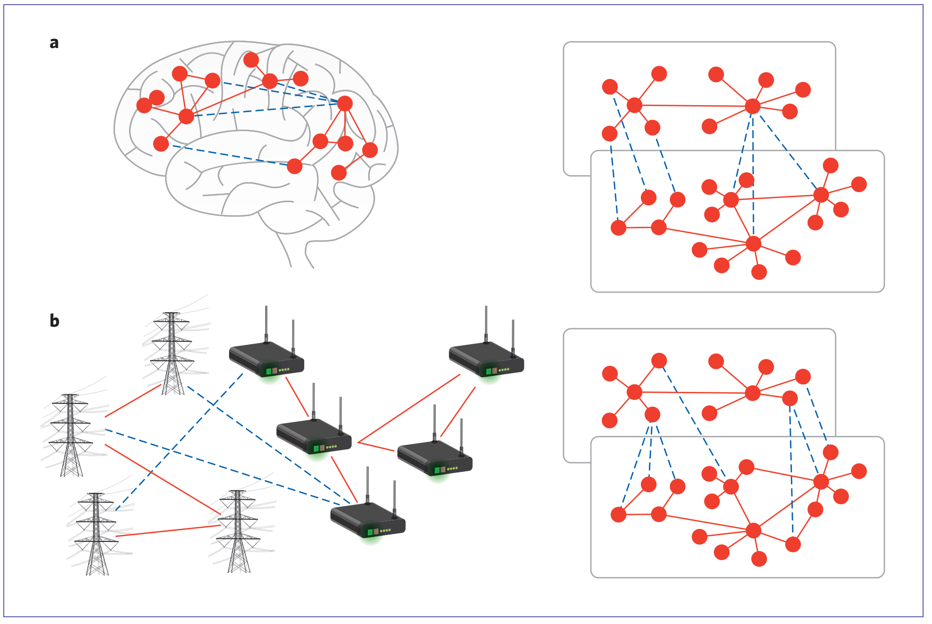

Figure 1 | Reis *et al.*[1] have shown that correlations between intra- (red) and interlayer (blue dotted) interactions influence the robustness of multilayer networks. **a**, In the brain, each network layer has multilayer assortativity and the hubs in each layer are also the nodes with more interlinks, so liaisons between layers are trustworthy. **b**, In complex infrastructures (such as power grids and the Internet), if the interlinks are random, the resulting multilayer network is affected by large cascades of failures[6], and liaisons can be considered dangerous.

layers, Reis *et al.*[1] revealed the important role of correlations in these networks of networks. Correlations are known to be ubiquitous in complex networks[9], but there can be different types in multilayer networks[10,11] and this study succeeded in determining the kind of correlation responsible for improved robustness.

This means we can distinguish between trustworthy and dangerous links — those that either improve or reduce the robustness of the multilayer networks (Fig. 1). Whereas random links between interdependent networks represent dangerous liaisons, enhancing the fragility of the entire system, the trustworthy interlinks between the networks are not random, but correlated in a specific way.

The multilayer network formed by scale-free networks exhibits improved robustness indicated by a smaller value of the percolation threshold (p_c) when two conditions are fulfilled. First, the interlinks between the layers must be such that the highly connected nodes, or hubs, of the single layers are also the nodes with more interlinks. And second, there must be multilayer assortativity. This means that for two layers, A and B, the hubs in layer A (layer B) are more likely to be linked with the nodes in layer B (layer A) that are connected with other hubs in layer B (layer A).

Reis *et al.*[1] analysed the percolation properties of two interacting scale-free networks, in which each node of one network could interact with several nodes in the other network. Two different dynamical rules determining the role of interlinks were considered: the conditional interaction and the redundant interaction. When the conditional interaction was taken into account, a node in layer A could not function, and was removed from the network, if all its connectivity with layer B was removed. A similar rule was adopted for nodes in layer B. When the redundant interaction was considered, a node in layer A was deemed functional if it belonged to the giant component in layer A, irrespective of the state of the linked nodes in the other layer, with a similar rule for nodes in layer B. The trustworthy interlinks were found to improve the robustness of multilayer networks with both conditional and redundant interactions.

The theory was validated by looking at the properties of two multilayer brain networks[12] reconstructed from functional magnetic resonance imaging studies. The first multilayer brain network was extracted from resting-state data and the second was obtained from dual-task data. The multilayer brain networks reconstructed from these studies organized into local brain networks formed by strongly correlated links connected by weak interlinks between the layers.

The two dynamical rules considered in the study[1] are both highly relevant to local brain networks. Tasks like processing different sensory features are independent of the interactions with other local networks (as in the redundant-interaction scenario), whereas processes like integrating perceptual information are possible only due to the coordinated activity of different local brain networks (as for conditional interactions). By analysing the brain as a multilayer network, Reis *et al.*[1] found that the interlinks are trustworthy, in agreement with their theoretical results and wider expectations related to the robustness of biological networks.

Reis and colleagues show that degree correlations might allow the realization of multilayer networks with improved robustness properties. This pivotal result solves the puzzle central to the existence of multilayer structure of biological networks. Moreover, a new question arises from this result: how we can we economically design robust multilayer networks of infrastructures or financial networks with trustworthy links? ❐

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