Comment on "Kinetic Roughening in Slow Combustion of Paper"

In a recent Letter, Maunuksela *et al.* present experiments on the combustion of paper [1]. They observe the roughening of the burning front and study the scaling properties of the correlation function of the height of the burning front. Maunuksela *et al.* find good agreement between the measured exponents characterizing the front roughening and the predictions of the Kardar-Parisi-Zhang (KPZ) equation [2], $\chi_{\rm KPZ} = \frac{1}{2}$ [χ is the roughness exponent which characterizes the scaling of the saturated height-height correlation function $G(r)$ with distance $G(r) \sim r^{2\chi}$]. Reference [1] also comments on the results of earlier experiments on paper burning by Zhang *et al.* [3] which measured $\chi = 0.71$, but offers no explanation for the difference in the measured value of the exponent.

Here, we show that the results of Maunuksela *et al.* [1] and of Zhang *et al.* [3] may be *both* understood under the framework of interface motion in disordered media [4–7].

For many experimental cases, the dominant source of noise in the dynamics is the *disorder in the medium* which is not time dependent. The universality classes for interface motion in disordered media have been identified and the values of the exponents are known, especially for $(1 + 1)$ dimensions [4–7]. One of the universality classes for interface motion in disordered media [7] can be described by the directed percolation depinning (DPD) model [4–6]. For a driving force F smaller than a critical value F_c, which defines the depinning transition, the interface eventually stops, and the scaling of $G(r)$ is characterized by an exponent $\chi_{\rm DPD} \approx 0.63$ [4]. If, on the other hand, $F > F_c$, the interface moves with an average velocity $v \sim (F - F_c)^\theta$ (where θ is a critical exponent), and $G(r)$ presents two scaling regimes in the steady state. For length scales *smaller* than the correlation length given by the disorder of the medium $\xi \sim (F - F_c)^{-\nu_\parallel}$ (where $\nu_\parallel$ is a critical exponent), $G(r)$ scales with an exponent $\chi'_{\rm DPD} \approx 0.75$, while for length scales *larger* than ξ, $G(r)$ scales with the KPZ exponent, $\chi_{\rm KPZ} = \frac{1}{2}$ [6].

To check that the results reported by Maunuksela *et al.* are consistent with the above theory, we digitize the data reported in [1] and plot it in Fig. 1 along with the predictions of the DPD model (see Refs. [4–6]). It is visually apparent that the experimental and theoretical data sets have the same two scaling regimes as discussed in Ref. [6]. Furthermore, the same theory may explain why Zhang *et al.* found no crossover to a KPZ dominated regime. Zhang's experiments were performed near the depinning transition, which implies that ξ was nearly as large as the system size. Thus, the quenched disorder dominates the scaling of $G(r)$ leading to the observation of only one scaling regime with a roughness exponent $\chi \approx \chi'_{\rm DPD}$. On the other hand, in the experiments reported by Maunuksela *et al.*, a shorter correlation length ξ is found, so that quenched disorder dominates only for small length scales where an exponent with values close to $\chi'_{\rm DPD}$ was measured. For length scales larger than ξ, time-dependent disorder dominates, and the KPZ exponent $\chi_{\rm KPZ}$ was measured, in agreement with the theory [6]. These results suggest that the DPD universality class may provide a compelling explanation of the experimental findings of Refs. [1,3].

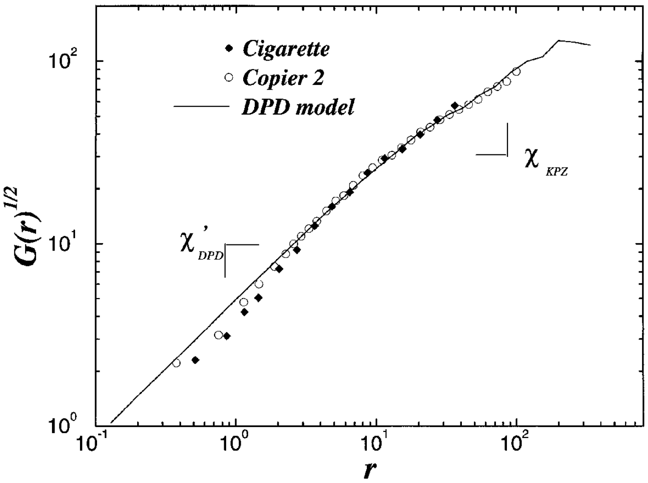

FIG. 1. Scaling of the height-height correlation function for the DPD model [6] and the data from the paper burning experiments of Ref. [1] for cigarette and copier papers. The results from the model were translated so as to make the crossover positions coincide. It is visually apparent that the two curves have the same two scaling regimes.

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