Solution

TEST # 3. PHYS 203. Chapters 8, 10, 11. Dec 8, 2022

NAME:

Rules for the exam:

1. The exam will start promptly at 5:00PM and will finish at 6:40PM.
2. No questions to the TA or Professor are allowed during the exam.
3. You can not leave the room under any circumstances.
4. You are allowed to have one cheat-sheet with no extra add-ons.
5. If you arrive to the exam after 5:15PM, you will not be allowed to do the exam.
6. You can use a calculator. No smartphones, mobile phones or any kind of computer devices are allowed during the exam.
7. No make-up under any circumstance.

1. Rotational Kinematics (30 points).

The wheels of a bicycle have an angular velocity of +15.0 rad/s. The brakes are applied and the wheels come to rest by making an angular displacement of +18.5 revolutions.

(a) (5 points). How much time did it take for the bike to come to rest?

(b) (10 points) Calculate the angular acceleration of the wheel in rad/s^2.

(c) (10 points) After coming to rest, the wheels accelerate with constant angular acceleration of +1.5rad/s^2. How much time did the wheels take to reach and angular velocity of 8 rad/s ?

(d) (5 points) Considering (c), how many revolutions does the wheel turn in this time interval?

$\omega_0 = 15\ rad/s$

$\theta_f = 18.5\ rev = 18.5\ rev \cdot \frac{2\pi}{rev} = 116.2\ rad$

(b) $\omega_f^2 - \omega_0^2 = -2\alpha\theta_f$

$\omega_f = 0 \qquad \left(15\frac{rad}{s}\right)^2 = 2 \times \alpha \times 116.2\ rad$

$\alpha = 1.0\ rad/s^2$

(a) $\omega_f = \omega_0 - \alpha t_f \Rightarrow 0 = 15\ rad/s - 1\frac{rad}{s^2} \cdot t_f$

$t_f = \frac{15\ rad/s}{1\ rad/s^2} = 15\ s$

(c) $\alpha = 1.5\ rad/s^2 \qquad \omega_0 = 0$

$\omega_f = 8\ rad/s$

$\Rightarrow 8\ rad/s = 0 + 1.5\ rad/s^2 \times t_f \rightarrow t_f = \frac{8\ rad/s}{1.5\ rad/s^2} = 5.3\ s$

(d) $\theta_f = \omega_0 t_f + \frac{1}{2}\alpha t_f^2 = \frac{1}{2} \times 1.5\ rad/s^2 \times (5.3\ s)^2 = 21\ rad$

($\omega_0 = 0$)

$\theta_f = 21\ rad \times \frac{1\ rev}{2\pi} = 3.3\ rev$

2. Elastic energy (35 points)

A 2 kg block is resting on horizontal frictionless surface and is attached to a horizontal spring whose spring constant is 150 N/m. The spring (with the block attached) is stretched initially to 6.5 cm and it is then released from rest.

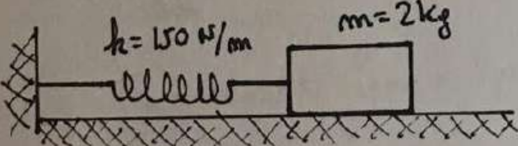
(a) (5 points) What is the amplitude of the resulting simple harmonic motion?

(b) (10 points) Determine the velocity (magnitude and direction as given by the sign) of the block when it comes back to the unstrained position after releasing it from rest.

(c) (5 points) What is the maximum compression of the spring?

(d) (10 points) Determine the two velocities of the block (magnitude and direction as given by the sign) when the spring is compressed at -3 cm.

(e) (5 points) Explain why you get two velocities in point (d).



(a) $A = 6.5$ cm

(b) $\frac{1}{2} \times 150 \frac{N}{m} \times (6.5 \times 10^{-2} m)^2 = \frac{1}{2} m v_f^2 = \frac{1}{2} 2 kg \times v_f^2$

$v_f = -0.56$ m/s

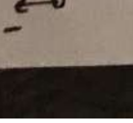
(c) $x_f = -6.50$ cm

(d) $\frac{1}{2} \times 150 \frac{N}{m} \times (6.5 \times 10^{-2} m)^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} k \times (3 \times 10^{-2} m)^2$

$v_f^2 = \frac{k}{m} \left[-(3 \times 10^{-2} m)^2 + (6.5 \times 10^{-2} m)^2 \right]$

$v_f = \pm 0.5$ m/s

(e)



3. Fluids (35 points)

A solid object made of pine with density $\rho_{pine} = 550\ \text{kg/m}^3$ measures $10m \times 10m \times 0.5m$. The density of water is $\rho = 1,000\ \text{kg/m}^3$.

(a) (5 points) Does the object float on water or not. Explain why?

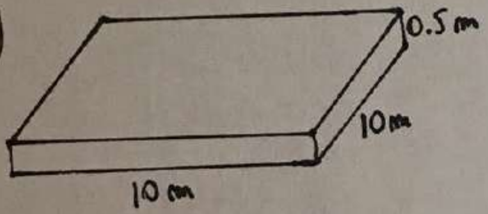
(b) (10 points) If it floats, what is the buoyancy force on the object?

(c) (10 points) If it floats, how much of the object will be under water?

(d) (5 points) If you double the density of the object, does it float or not? Explain.

(d) (5 points) If the volume of the object is reduced to $5m \times 5m \times 0.5m$, calculate how much of the object will be under water.

(a)



yes $\rho_{pine} < \rho_{water}$

(b) $F_B = W_{obj} = \rho_{pine} \times V_{pine} \times g = 550 \frac{kg}{m^3} \times 10m \times 10m \times 0.5m \times 9.8 m/s^2 =$

$F_B = 269.5 \times 10^3 N$

(c) $\therefore F_B = 269.5 \times 10^3 N = \rho_{water} \times 10m \times 10m \times h \times g =$

$= \rho_{pine} \times 10m \times 10m \times 0.5m \times g$

$\Rightarrow h = \frac{\rho_{pine}}{\rho_{water}} \times 0.5m = \frac{550}{1000} \times 0.5m = 0.275\ m.$

(c) → sinks

(d) $h = 0.275\ m$